

Deblurring with Rank-structured Inverse Approximations

Mary Hudachek-Buswell*
Georgia State University
Clayton State University

Catherine Matos†
Clayton State University

Michael Stewart‡
Georgia State University

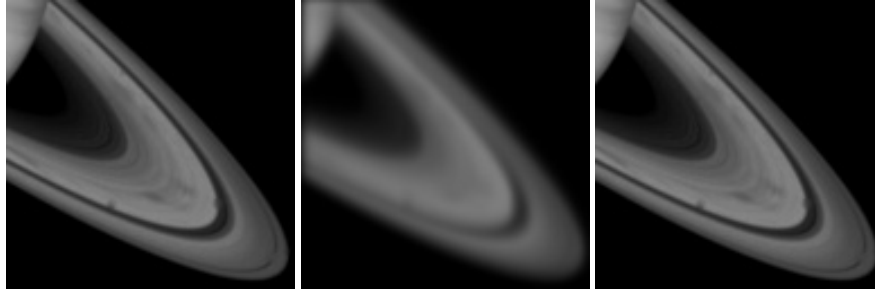


Figure 1: (a) The original satellite image of Saturn’s rings. (b) The image blurred by a skewnormal PSF to model atmospheric turbulence. (c) The deblurred image from proposed rank-structured inverse approximation method.

Abstract

In this presentation, the restoration of images blurred by atmospheric turbulence is examined. The proposal uses a new class of approximations to blurring operators representing Gaussian blur. The Toeplitz matrix representing the blur is transformed into a Cauchy-like (CL) matrix using the FFT. In addition to the CL structure, the transformed matrix has a rank structure. In particular, the off-diagonal blocks have low rank. This class of matrices can be approximated quickly, and the structure can be exploited for fast image restoration.

Keywords: Deblurring, Toeplitz matrices, structured matrices, rank-structured matrices, superfast algorithm

1 Introduction

A satellite image is degraded by atmospheric turbulence and additive noise. Direct inverse methods to restore the image use a point spread function, PSF, to model the blur. The matrix equation $Ax = b$ represents the blurring, where A is the blur matrix, x is the restored image, and b is the distorted blurred image. Direct inverse methods either explicitly invert the matrix A or some approximation to A to obtain $x = A^{-1}b$. Such methods can be sensitive to noise because the matrix A is often severely ill-conditioned. Techniques for addressing this difficulty include regularization, and iterative methods such as conjugate gradient (CG) and generalized minimal residual (GMRES). Approximations to the matrix A can be exploited in all of these approaches. This work approximates A with a rank-structured matrix [Chandrasekaran et al. 2007]. Such approximations can be efficiently computed, efficiently inverted, and can be applied as a preconditioner to enhance the convergence of an iterative method [Nagy et al. 1996].

2 Methods and Implementation

The blur used in this work is a skewnormal distribution [Hansen and Jensen 2008]. The FFT is used to transform the blur matrix

into a CL matrix with rank structure. In particular, the transformed $n \times n$ matrix A when being partitioned as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

has the property that each of the off-diagonal blocks, A_{12} and A_{21} , has low rank and can be represented by $O(n)$ parameters. This holds for any such partition and, applied recursively, leads to a representation of the entire matrix in terms of $O(n)$ parameters. A fast algorithm is used to extract the rank structure in an exploitable form [2007]. The new structure allows for an $O(n)$ solution to the approximate system for deblurring. The structured matrix is used in an approximate inverse method and as a basis for a preconditioned iterative method.

3 Comments and Results

Inverse filtering or preconditioned iteration techniques typically use circulant approximations to the blur matrix in order to exploit the FFT in multiplication by the circulant matrix. In this work, a broader class of approximations is considered to yield better inverse approximations and faster convergence. Of twenty-two surveyed, 91% chose the deblurred image by rank-structured inverse approximation as opposed to restored images without the proposed method. Results of using the approximation in an iterative reconstruction are shown above in Figure 1.

References

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*e-mail: MaryHudachek-Buswell@clayton.edu

†e-mail:cmatos@clayton.edu

‡e-mail:mastewart@gsu.edu