

### Section 6.3: Proving Trigonometric Identities

#### Key points:

- Know the trigonometric identities given in **Section 6.1** and **Section 6.2!** These will be used extensively in the proofs.
- There are two basic methods to start:
  - (1) Start with the more complicated side and obtain the other side
  - (2) Work with each side separately to obtain a common expression
- Convert *everything* to sines and cosines
- Multiply rational expressions by a special form of 1
- Carry out algebraic manipulations
- TRY SOMETHING!

EXAMPLE 1. Prove:  $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$ .

**proof:**

$$\begin{aligned} \text{LHS} &= (\sin \theta + \cos \theta)^2 = (\sin \theta + \cos \theta)(\sin \theta + \cos \theta) \\ &= \sin^2 \theta + \sin \theta \cos \theta + \sin \theta \cos \theta + \cos^2 \theta \\ &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ &= 1 + \sin 2\theta \\ &= \text{RHS} \quad \square \end{aligned}$$

EXAMPLE 2. Prove:  $\frac{\cos(u-v)}{\cos u \sin v} = \tan u + \cot v$ .

**proof:**

$$\begin{aligned} \text{LHS} &= \frac{\cos(u-v)}{\cos u \sin v} = \frac{\cos u \cos v + \sin u \sin v}{\cos u \sin v} \\ &= \frac{\cancel{\cos u} \cos v}{\cancel{\cos u} \sin v} + \frac{\sin u \cancel{\sin v}}{\cos u \cancel{\sin v}} \\ &= \frac{\cos v}{\sin v} + \frac{\sin u}{\cos u} \\ &= \cot v + \tan u \\ &= \text{RHS} \quad \square \end{aligned}$$