

Section 5.6: Graphs of Transformed Sine and Cosine

Key points:

- Graph the transformed sine and cosine functions

$$y = A \sin(Bx - C) + D$$

$$y = A \cos(Bx - C) + D$$

using technology.

- Determine algebraically the amplitude, period, phase shift, vertical shift, and relative extrema for these graphs.
- Graph sums of functions: pages 518-20.
- Graph damped oscillations: pages 520-1.

Transformations

The graphs of

$$y = A \sin(Bx - C) + D$$

$$y = A \cos(Bx - C) + D$$

are the graphs of the basic functions

$$y = \sin(x)$$

$$y = \cos(x),$$

respectively, under various transformations: horizontal shifts left or right, vertical shifts up or down, horizontal stretching or shrinking, and vertical stretching or shrinking. It is very helpful to understand the basic graphs (see **Section 5.5**) before moving on to the transformations of these graphs.

One of the better ways to understand this section is to graph lots of functions and *see what happens*.

There is a very good summary of the transformations on page 516.

EXAMPLE 1. Find the period, phase shift, amplitude, and vertical shift for the graph of

$$f(x) = -2\sin(4x + \pi) + 3.$$

Then determine the maximum and minimum values of f and graph f .

Solution: The period is

$$\text{period} = \left| \frac{2\pi}{B} \right| = \left| \frac{2\pi}{4} \right| = \frac{\pi}{2}.$$

The phase shift is

$$\text{phase shift} = \frac{C}{B} = \frac{-\pi}{4} = -\frac{\pi}{4}.$$

The amplitude is

$$\text{amplitude} = |A| = |-2| = 2.$$

The vertical shift is

$$\text{vertical shift} = D = 3.$$

The maximum value of f is

$$\text{max value} = \text{vertical shift} + \text{amplitude} = 3 + 2 = 5$$

and the minimum value of f is

$$\text{min value} = \text{vertical shift} - \text{amplitude} = 3 - 2 = 1.$$

Use technology to graph f .