Section 5.2: Applications of Right Triangles

Key points:

- Solve right triangles
- Solve applied problems involving right triangles
- Angles of elevation/depression
- Bearing

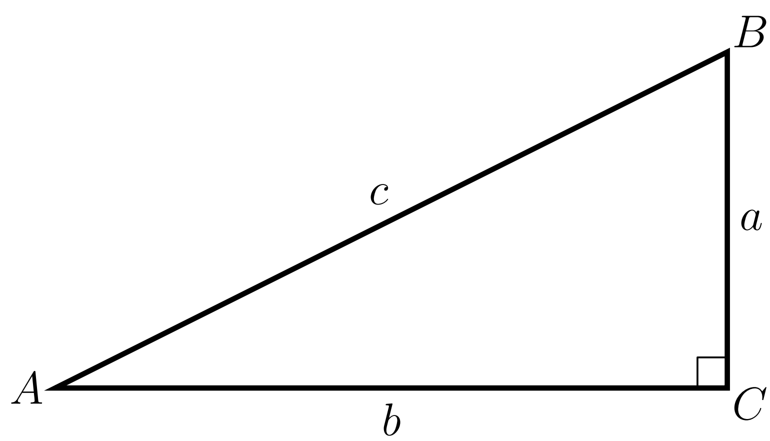


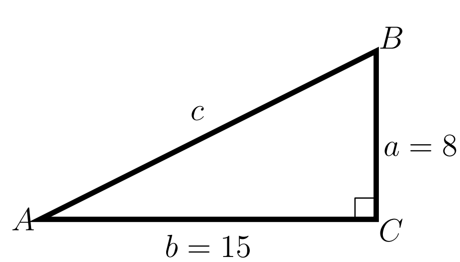
FIGURE 1. A standard right triangle has legs a, b and hypotenuse c. Angle A is across from side a, angle B is across from side b, and angle C (the right angle) is across from side c. Note that A and B are complementary angles.

Tools for Solving Right Triangles:

(1) Start by drawing a picture that is labeled with the given information.
(2) Find any missing angles and sides using the trig definitions/ratios or the Pythagorean Theorem.
(3) Avoid using an approximation at an intermediate step; solve the triangle by using the given information if at all possible.
(4) Summarize the results.

EXAMPLE 1. In a certain right triangle, $a = 8$ and $b = 15$. Solve the triangle.

Solution: First we draw the right triangle using the given information:



Side c can be found using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 = c^2 \Longrightarrow c &= \sqrt{a^2 + b^2} \\ &= \sqrt{8^2 + 15^2} = 17. \end{aligned}$$

Angle A can be found in many ways, one of which uses the tangent function:

$$\begin{aligned} \tan A = \frac{8}{15} \Longrightarrow A &= \tan^{-1}(\frac{8}{15}) \\ &\approx 28.07^\circ. \end{aligned}$$

Similarly,

$$\begin{aligned} \tan B = \frac{15}{8} \Longrightarrow B &= \tan^{-1}(\frac{15}{8}) \\ &\approx 61.93^\circ. \end{aligned}$$

We could have also used the fact that angle B and angle A are complementary:

$$B = 90^\circ - A \approx 61.93^\circ,$$

but we would have used an approximation at an intermediate step. (Here, we got the same approximate answers anyway, but this may not always happen due to rounding errors!)

Thus,

$$\begin{aligned} &A \approx 28.07^\circ \qquad && a = 8 \\ &B \approx 61.93^\circ && b = 15 \\ &C = 90^\circ && c = 17 \end{aligned}$$

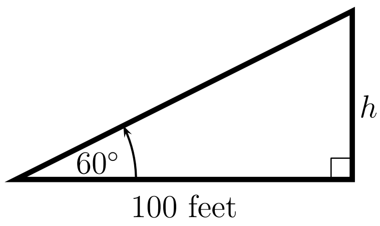
Angles of Elevation and Depression

Angles of elevation are usually measured from the ground upward. Angles of depression are usually measured from some object that is in the air downward toward the ground.

For example, if a person were standing on the ground looking upward at a bird in a tree, then we would refer to this as an angle of elevation. If a person were in a airplane looking out the window at a car on the ground, then we would refer to this as an angle of depression.

Example 2. Jenny walks 100 feet from the base of a tree and estimates the angle of elevation to the top of the tree as 60°. Find the approximate height of the tree.

Solution: First we draw the right triangle using the given information, letting h represent the height of the tree:



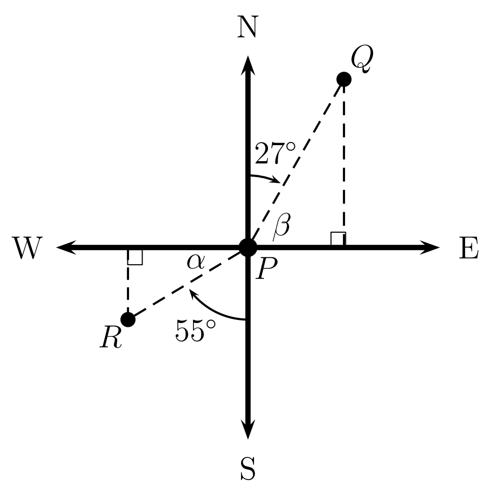
Then

$$\tan 60^\circ = \frac{h}{100} \Longrightarrow h = 100 \tan 60^\circ$$
$$= 100\sqrt{3} \approx 173,$$

so that the tree is approximately 173 feet tall.

Bearing (from a point)

Bearing is always measured from some point P to some other point. We simply draw a compass centered at P with N, E, S, W appropriately labeled, as in the following figure:



Example 3. (a) The bearing from P to Q is N27°E. (b) The bearing of R from P is S55°W.

Example 4. (a) What is the measure of β? (b) What is the measure of α?

Solution: This is easy: $\beta = 90^\circ - 27^\circ = 63^\circ$ and $\alpha = 90^\circ - 55^\circ = 35^\circ$.