Section 5.1: Trigonometric Functions of Acute Angles

Key points:

- Determine the 6 trig ratios for an acute angle of a right triangle
- Determine the exact values of the trig functions at 30°, 45°, and 60°
- Convert angles between decimal degrees and $D°M'S''$
- Find approximate values of the trig functions at any acute angle
- Given a trig function value at an acute angle, find the acute angle
- Cofunction Identities

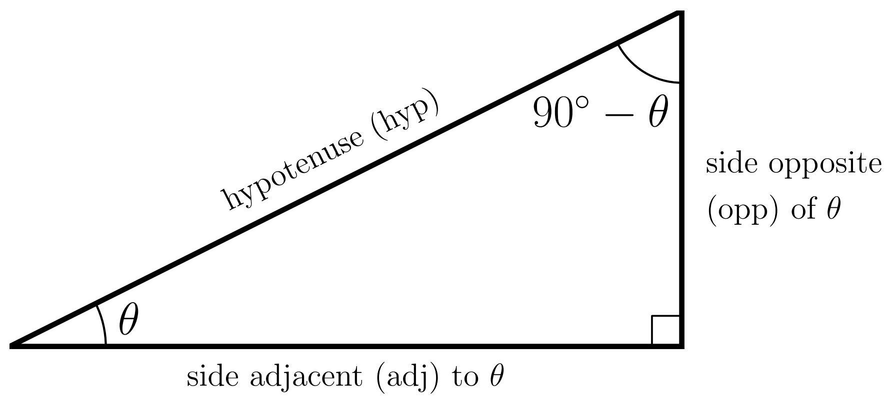


FIGURE 1. We will use a right triangle such as this *very* often!

The Trig Ratios

sine : $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

cosecant : $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$

cosine : $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

secant : $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}}$

tangent : $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\text{opp}}{\text{adj}}$

cotangent : $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\text{adj}}{\text{opp}}$

Facts: Given a right triangle with acute angle θ:

(1) If $\theta < 45°$, then the side opposite θ will be the shorter leg and the side adjacent to θ will be the longer leg.
(2) If $\theta = 45°$, then the side opposite θ and the side adjacent to θ will have equal length.
(3) If $\theta > 45°$, then the side opposite θ will be the longer leg and the side adjacent to θ will be the shorter leg.

Fact: Given two *similar* right triangles, one with acute angle α and one with acute angle β, if $\alpha = \beta$, then the values of the trig functions at α are equal to the values of the trig functions at β. The converse is also true; that is, given two right triangles, one with acute angle α and one with acute angle β, if the values of the trig functions at α are equal to the values of the trig functions at β, then $\alpha = \beta$ and the triangles are similar.

Exact Values at $30°$, $45°$, and $60°$

One way to remember the exact values of the 6 trig functions at $30°$, $45°$, and $60°$ is to draw a $30° - 60° - 90°$ triangle and a $45° - 45° - 90°$ triangle, as seen in Figure 2 and Figure 3.

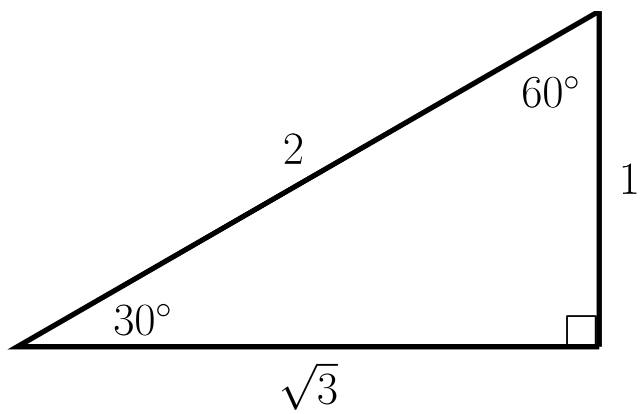


Figure 2. Drawing a $30° - 60° - 90°$ triangle helps to remember the exact values of the trig functions at $30°$ and $60°$.

$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \sqrt{3}$$

$$\csc 30° = \frac{1}{\sin 30°} = 2 \qquad \csc 60° = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec 30° = \frac{1}{\cos 30°} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \sec 60° = \frac{1}{\cos 60°} = 2$$

$$\cot 30° = \frac{1}{\tan 30°} = \sqrt{3} \qquad \cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

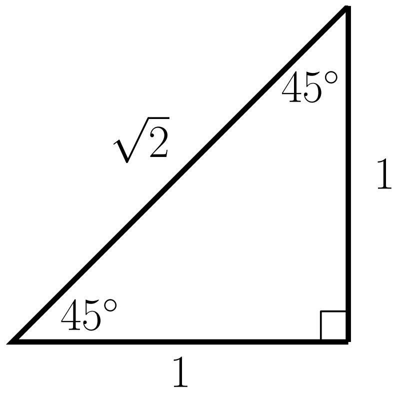


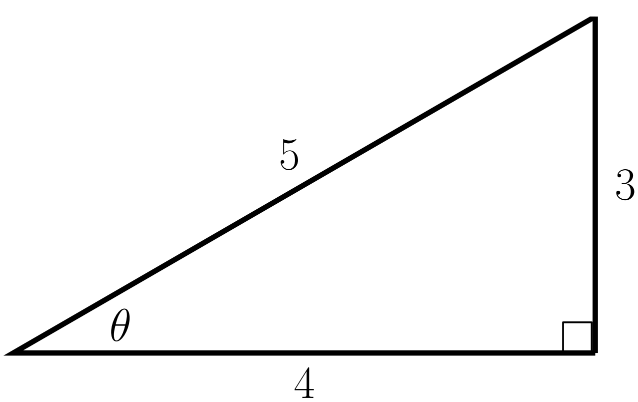
FIGURE 3. Drawing a $45°-45°-90°$ triangle helps to remember the exact values of the trig functions at $45°$.

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc 45° = \frac{1}{\sin 45°} = \sqrt{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec 45° = \frac{1}{\cos 45°} = \sqrt{2}$$

$$\tan 45° = 1 \qquad \cot 45° = \frac{1}{\tan 45°} = 1$$

EXAMPLE 1. Given the following right triangle, find the values of the 6 trig functions at θ:



Solution:

$$\sin\theta = \frac{3}{5} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{5}{3}$$

$$\cos\theta = \frac{4}{5} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$$

$$\tan\theta = \frac{3}{4} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$$

EXAMPLE 2. For the triangle given in EXAMPLE 1, is $\theta < 45°$, $\theta = 45°$, or $\theta > 45°$?

Solution: In EXAMPLE 1, θ is opposite the shorter leg, so $\theta < 45°$.

Converting Between Decimal Degrees and $D°M'S''$

$$1 \text{ degree} = 60 \text{ minutes}$$
$$1° = 60'$$

$$1 \text{ minute} = 60 \text{ seconds}$$
$$1' = 60''$$

$$1 \text{ degree} = 60 \times 60 \text{ seconds}$$
$$1° = 3600''$$

EXAMPLE 3. Convert $34°25'45''$ to decimal degrees.

Solution: $34°25'45'' = 34° + \dfrac{25}{60}^{\circ} + \dfrac{45}{3600}^{\circ} \approx 34.429°$

Approximate Values of Trig Functions

EXAMPLE 4. Approximate to 4 decimal places: $\sin 24°$.

Solution: $\sin 24° \approx 0.4067$

EXAMPLE 5. Approximate to 4 decimal places: $\sec(12°35'42'')$.

Solution: $\sec(12°35'42'') = \dfrac{1}{\cos(12°35'42'')} = \dfrac{1}{\cos(12.595°)} \approx 1.0247$

Finding Acute Angles

Review: Section 4.1: Inverse Functions, p. 348. If f "does" something, then f^{-1} "undoes" it.

- $\sin^{-1}(x)$ is read "the arcsine of x".
- $\cos^{-1}(x)$ is read "the arccosine of x".
- $\tan^{-1}(x)$ is read "the arctangent of x".

Basically, if

$$\sin(\text{some acute angle}) = \text{ some positive number},$$

then

$$\sin^{-1}(\text{some positive number}) = \text{ some acute angle}.$$

Similarly for cosine and tangent.

EXAMPLE 6. Since $\sin 30° = \frac{1}{2}$, we have $\sin^{-1}(\frac{1}{2}) = 30°$. Since $\cos 45° = \frac{\sqrt{2}}{2}$, we have $\cos^{-1}(\frac{\sqrt{2}}{2}) = 45°$.

EXAMPLE 7. Find the acute angle, to the nearest degree, whose cosine is 0.766.

Solution: We need to find θ such that $\cos\theta = 0.766$; that is,

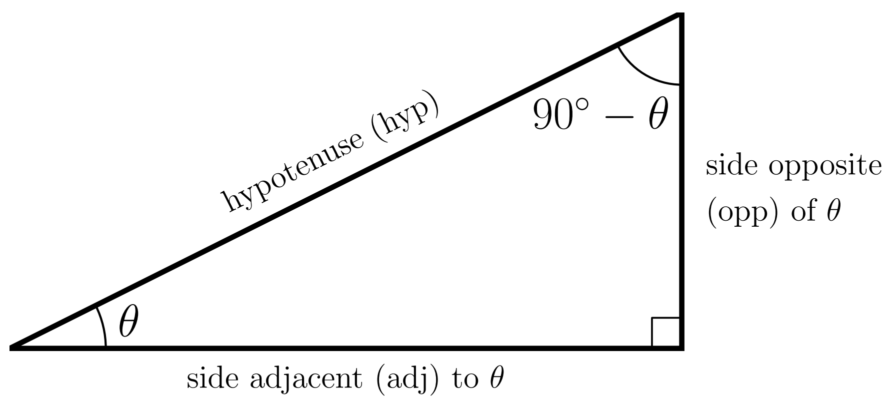
$$\theta = \cos^{-1}(0.766) \approx 40°.$$

EXAMPLE 8. Suppose β is an angle in Quadrant I with $\tan\beta = 4.91516$. Find β to the nearest tenth of a degree.

Solution: Since $\tan\beta = 4.91516$, we have

$$\beta = \tan^{-1}(4.91516) \approx 78.5°.$$

Cofunction Identities



For any right triangle with acute angle θ:

$$\sin\theta = \cos(90° - \theta) \qquad \cos\theta = \sin(90° - \theta)$$

$$\tan\theta = \cot(90° - \theta) \qquad \cot\theta = \tan(90° - \theta)$$

$$\sec\theta = \csc(90° - \theta) \qquad \csc\theta = \sec(90° - \theta)$$