Section 4.6: Exponential Growth and Decay

Key points:

- Solve applied problems involving exponential growth and decay.
- Use the Continuously Compounded Interest Formula.

Growth Model

Exponential growth is modeled by

$$P(t) = P_0 e^{kt}, \quad \text{where} \quad k > 0.$$

In this formula,

- $P(t)$ is the amount after time t
- P_0 is the initial amount or starting value
- e is Euler's number
- k is the growth rate, as a decimal
- t is time, in appropriate units
- Growth Model: $P(t)$ gets larger as t gets larger

The time required for the initial amount to double in size is called the **doubling time**, which is given by

$$\text{doubling time } = \frac{\ln 2}{k}.$$

Continuously Compounded Interest

This is a particular type of exponential growth, usually given by

$$P(t) = P_0 e^{kt} \quad \text{becomes} \quad A = Pe^{rt},$$

where

- A is the amount in the account or amount owed at time t
- P is the principal or initial amount invested/borrowed
- e is Euler's number
- r is the interest rate, as a decimal
- t is time, in years

Decay Model

Exponential decay is modeled by

$$P(t) = P_0 e^{-kt}, \quad \text{where} \quad k > 0.$$

In this formula

- $P(t)$ is the amount after time t
- P_0 is the initial amount or starting value
- e is Euler's number
- k is the decay rate, as a decimal
- t is time, in appropriate units
- Decay Model: $P(t)$ gets smaller as t gets larger

The time required for the only half a substance to remain is called the **half-life**, which is given by

$$\text{half-life} = \frac{\ln 2}{k}.$$

Note: Carbon-14 (C-14) is used in carbon dating. The half-life is about 5750 years. This means that the decay rate for C-14 is about 0.012% per year, that is $k \approx 0.00012$.