Section 4.2: Exponential Functions

Key points:

- Graph exponential functions.
- Solve applied problems involving exponential functions and their graphs.
- Use the Compound Interest Formula.

Exponential Function

An **exponential function** in base a is given by

$$f(x) = a^x,$$

where x is any real number and $a > 0$, but $a \neq 1$.

The Number *e*

The number e is **Euler's number**, given by

$$e \approx 2.718281828459045 23 \ldots$$

It is a nonterminating, non-repeating decimal (similar to π). Just like π, Euler's number is a very important number that arises in many fields of mathematics.

The Natural Exponential Function

The **natural exponential function** is

$$f(x) = e^x,$$

where e is Euler's Number.

Properties of Exponential Functions

Suppose $f(x) = a^x$, where $a > 0$ and $a \neq 1$ is an exponential function. Then the following are true:

- Domain: $(-\infty, \infty)$.
- Range: $(0, \infty)$.
- An exponential function is **increasing** if $a > 1$.
- An exponential function is **decreasing** if $0 < a < 1$.
- The y-intercept is the point $(0, 1)$.
- The horizontal asymptote is the x-axis,that is, the line $y = 0$.
- All exponential functions are $1-1$. Their inverses are called **logarithmic functions**. See **Section 4.3**.

Compound Interest Formula

The compound interest formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

is an exponential function. In this formula

- A is the total amount of money
- P is the principal, or starting amount
- r is the interest rate, as a decimal
- n is the number of times interest is compounded per year
- t is time, in years

Some common numbers of compounding are

- annually: $n = 1$
- semiannually: $n = 2$
- quarterly: $n = 4$
- monthly: $n = 12$
- daily: this can vary, usually either $n = 360$ or $n = 365$