## Section 3.2: Graphing Polynomial Functions

**Key points:**

- A polynomial with degree $n$ has **at most** $n-1$ turning points. A turning point is a relative maximum or a relative minimum: p. 270.
- Must know how to graph a polynomial: p. 272.
- Be able to state and use The Intermediate Value Theorem: p. 276.

**To graph a Polynomial**:

1. Use the Leading Term Test for End Behavior to determine what the graph will "look like" on the ends.
2. Determine the multiplicity of real roots and describe what the graph will "look like" near the $x$-intercepts.
3. Find some test points, including the $y$-intercept.
4. Sketch the graph.
5. Use technology to graph the polynomial.
6. Find/approximate the extrema (relative max/min, turning points).
7. Determine intervals of increasing or decreasing behavior.

### The Intermediate Value Theorem

Let $f(x)$ be a polynomial with REAL coefficients and suppose $a, b$ are real numbers with $a \neq b$. If $f(a)$ and $f(b)$ have different signs (one is positive and one is negative), then $f$ has a real zero/root between $x = a$ and $x = b$. In other words, there is some real value $x = c$ between $a$ and $b$ such that $f(c) = 0$.

The IVT can be used to approximate a real root and may be helpful in finding the location of an $x-$intercept.

If $f(a)$ and $f(b)$ have the same sign (both positive or both negative), then the IVT does not apply.

**Example 1.** Suppose $f(x) = 3x^2 + 5x - 4$. Show that $f$ has a real root between $a = 0$ and $b = 1$.

It is easy to see that

$$f(a) = f(0) = -4 < 0$$

and

$$f(b) = f(1) = 4 > 0.$$

Since $f(0)$ is negative and $f(1)$ is positive, the Intermediate Value Theorem says that the quadratic function $f$ has a real root/zero between $x = 0$ and $x = 1$. Graph $f$ and verify that this is true. Can you find two other consecutive integers $a$ and $b$ such that $f$ has a real root between $a$ and $b$? Guess the values of the two roots of the function $f$.