

Section 2.4: Graphs of Quadratic Functions (Parabolas)

Key points:

- Find the vertex of a parabola.
- Find the standard form of a quadratic function.
- Determine whether a parabola opens upward (has a relative/global minimum) or opens downward (has a relative/global maximum).
- Find the intervals of increasing or decreasing behavior.
- Find the domain and range of a quadratic function.
- Find the axis of symmetry.
- Find the x - and y -intercepts for a quadratic function.
- Graph quadratic functions.
- Solve applied problems involving maximum and minimum function values.

Quadratic Function: $f(x) = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$. The graph of a quadratic function is a *parabola*.

Vertex: The vertex is the point $(h, k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$:

- the x -coordinate of the vertex is $h = \frac{-b}{2a}$
- the y -coordinate of the vertex is $k = f(h)$

Standard Form of a Quadratic Function: $f(x) = a(x - h)^2 + k$ where a is the leading coefficient and (h, k) is the vertex. When written in this way, it should be clear how transformations are used to change the graph of the basic function $y = x^2$ into the graph of a more “complicated” function like $y = -3(x + 2)^2 - 4$.

Relative/Global Extrema:

- If $a > 0$, then the parabola opens *upward* and the vertex is the lowest point on the graph. The **relative/global minimum** OCCURS at $x = h$ and has VALUE $f(h) = k$.
- If $a < 0$, then the parabola opens *downward* and the vertex is the highest point on the graph. The **relative/global maximum** OCCURS at $x = h$ and has VALUE $f(h) = k$.

Increasing and Decreasing Behavior:

- If $a > 0$, the parabola opens upward; the function changes from decreasing to increasing at the x -coordinate of the vertex, so that the quadratic function is decreasing on $(-\infty, h)$ and increasing on (h, ∞) .
- If $a < 0$, the parabola opens downward; the function changes from increasing to decreasing at the x -coordinate of the vertex, so that the quadratic function is increasing on $(-\infty, h)$ and decreasing on (h, ∞) .

Range of a Quadratic Function:

- If $a > 0$, the parabola opens upward, and the vertex is the lowest point on the graph. The range will “start” at the y -coordinate of the vertex—the range will be $[k, \infty)$.
- If $a < 0$, the parabola opens downward, and the vertex is the highest point on the graph. The range will “end” at the y -coordinate of the vertex—the range will be $(-\infty, k]$.

Domain of a Quadratic Function: The domain of all quadratic functions is **all real numbers**, $\mathbb{R} = (-\infty, \infty)$.

Axis of Symmetry: “Splits the graph in half”; A vertical line through the vertex; The **line** $x = h$.

The y -intercept: The y -intercept for any function can be found by substituting $x = 0$ into the function. The y -intercept of any function f has coordinates $(0, f(0))$.

The x -intercepts: The x -intercepts for any function can be found by setting $f(x) = 0$ and solving for x . Any *REAL* root/zero/solution $x = c$ provides an x -intercept for the graph at the point $(c, 0)$. Review the notes from

Section 2.3: Quadratic Equations, Functions, and Models concerning the *discriminant*.

Graphing Quadratic Functions: The graph will be a parabola. Combining the above ideas should provide enough information to sketch a graph.

Solving Applied Problems: Please work the word problems on pp. 227-8: 35, 37, 39, 40, 41, 42, 43, 45, 49.