Section 2.3: Quadratic Equations, Functions, and Models

Key points:

- Find zeros of quadratic functions.
- Solve quadratic equations: (1) by factoring, (2) by completing the square, or (3) by using the quadratic formula.
- Use the discriminant to classify solutions.
- Solve equations that are reducible to quadratic.
- Solve applied problems using quadratic equations.
- Given an application, find the domain of the associated quadratic function.

Quadratic Equation: $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

Quadratic Function: $f(x) = ax^2 + bx + c$; the graph is a **parabola** that opens *upward* if $a > 0$ and opens *downward* if $a < 0$.

Zeros of a Function: Any value $x = c$ that gives $f(c) = 0$; Any value of x that makes the functions' value equal 0.

- The zeros of *any* function f are found by setting $f(x) = 0$ and solving for x. The *solutions* or *roots* of the equation are those values $x = c$ that give $f(c) = 0$.
- If a solution/root/zero $x = c$ is a *REAL NUMBER*, then the point $(c, 0)$ is an ***x*-intercept** for the graph.
- The graph of a quadratic function may be helpful in locating the x-intercepts, which in turn give information about the real solutions. Do not think, however, that the graph *tells* you what the solutions to an equation are–you need the algebra for that!
- Roots and factors "go together"; if $x = c$ is a root of f, then $(x - c)$ is a factor of f; if $(x - c)$ is a factor of f, then $x = c$ is a root of f.

Factoring: Review **Section R.3** and **Section R.4**.

Completing the Square: See "To solve a quadratic equation by completing the square" on p. 205.

The Quadratic Formula: The solutions of *every* quadratic equation of the form

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic formula can be used to solve *every* quadratic equation, but it may not be the easiest way!

The Discriminant: The number $b^2 - 4ac$; the number under the radical in the quadratic formula.

- If $b^2 - 4ac < 0$, then there are **two non-real solutions**; they will be complex conjugates; the graph will have no x-intercepts.
- If $b^2 - 4ac = 0$, then there is **one real solution** $x = \frac{-b}{2a}$; it is called a **double root**; the graph will *touch* (is tangent to) the x-axis at the x-intercept.
- If $b^2 - 4ac > 0$, then there are **two distinct real solutions**; the graph will *cross* the x-axis at two different points; there are two distinct x-intercepts.

Applications: Know how to find the perimeter and area of common geometric figures: triangle, square, rectangle, parallelogram, and circle. Know how to find the volume of rectangular solids and cubes. These can be found in the back of the book under **Geometry**.

It is important to understand the inherent domain restrictions for any functions that arise from applications. Remember:

All dimensions must be positive real numbers!