Section 2.1: Linear Equations, Functions, and Models

Key points:

- Solve linear equations.
- Solve a formula for a variable.
- Find zeros of linear functions.
- Solve applied problems using linear models.

Linear Equations

Linear equation: A linear equation can be written as

$$mx + b = 0,$$

where m and b are real and $m \neq 0$.

Any value of x that satisfies an equation is called a **solution** or **root** of the equation.

Example 1. Solve $5x + 3 = 18$.

$$\begin{aligned} 5x + 3 &= 18 \\ 5x &= 15 \\ x &= 5 \end{aligned}$$

Example 2. Solve $-3(t+1) - 4 = 4t + 5(t-2)$.

$$\begin{aligned} -3(t+1) - 4 &= 4t + 5(t-2) \\ -3t - 3 - 4 &= 4t + 5t - 10 && \text{(distribute)} \\ -3t - 7 &= 9t - 10 && \text{(combine like terms)} \\ -7 &= 12t - 10 && \text{(add } 3t \text{ to both sides)} \\ 3 &= 12t && \text{(add 10 to both sides)} \\ t &= \frac{3}{12} = \frac{1}{4} && \text{(divide both sides by 12)} \end{aligned}$$

Example 3. Solve $Ax + By + C = 0$ for y.

$$\begin{aligned} Ax + By + C &= 0 \\ By &= -Ax - C \\ y &= \frac{-Ax - C}{B} = -\frac{A}{B}x - \frac{C}{B} \end{aligned}$$

Zeros of Linear Functions

Suppose $f(x)$ is a function. Any value $x = c$ that gives $f(c) = 0$ is called a **zero** of the function f. To find the zeros of any function, set the function equal to 0 and solve for x.

If c is a *real number* such that $f(c) = 0$ (that is, c is a **real zero** of $f(x) = 0$) then the point $(c, 0)$ is an ***x*-intercept** for the graph of f.

Example 4. Find the zero of $f(x) = -3x + 9$.

Set $f(x) = 0$ and solve for x:

$$\begin{aligned} -3x + 9 &= 0 \\ -3x &= -9 \\ x &= 3. \end{aligned}$$

The zero or root of f is $x = 3$, which is a real number. Then the x-intercept for the graph of f is the point $(3, 0)$. Since f is a linear function, the graph is a line. The slope is $m = -3$, so the graph of f is *decreasing* over the entire domain. The y-intercept is the point $(0, 9)$.

Example 5. Find the x-intercept for the graph of $g(x) = \frac{2}{5}x + \frac{8}{25}$.

If g has a real root, then that root will provide information about the x-intercept:

$$\begin{aligned} \frac{2}{5}x + \frac{8}{25} &= 0 \\ \frac{2}{5}x &= -\frac{8}{25} \\ x &= -\frac{8}{25} \cdot \frac{5}{2} = -\frac{4}{5}. \end{aligned}$$

Since the zero of g is $x = -\dfrac{4}{5}$, which is real, the graph of g has an x-intercept at the point $\left(-\dfrac{4}{5}, 0\right)$. The slope of this linear function is $m = \dfrac{2}{5}$, so the graph of g is *increasing* over the entire domain, and the y-intercept is $\left(0, \dfrac{8}{25}\right)$.

Example 6. Solve Exercise 25, page 188.

Example 7. Solve Exercise 36, page 189.

Example 8. Solve Exercise 53, page 190.

Example 9. Solve Exercise 30, page 188.