Section 1.5: More on Functions

Key points:

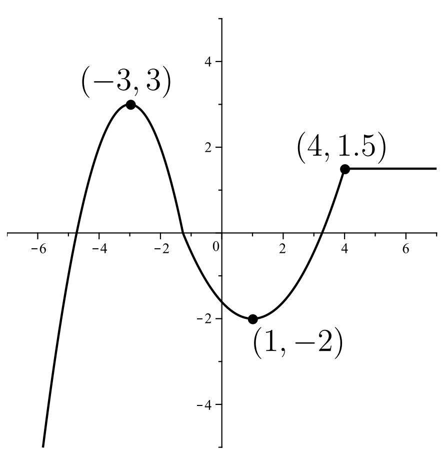
- Given a graph or equation of a function, determine intervals of increasing, decreasing, or constant behavior.
- Estimate relative extrema.
- Use a function to model an application and find the domain of the function.
- Graph piecewise functions.

Increasing, Decreasing, or Constant Behavior

- The **intervals of increasing behavior** are the values of x for which the function values are getting larger.
- The **intervals of decreasing behavior** are the values of x for which the function values are getting smaller.
- The **intervals of constant behavior** are the values of x for which the function values stay the same.

These are **always** given as open intervals!

Example 1. The graph of some function f is given below.



domain: all reals $= (-\infty, \infty)$

range: $(-\infty, 3]$

increasing: $(-\infty, -3) \cup (1, 4)$

decreasing: $(-3, 1)$

constant: $(4, \infty)$

Relative Extrema

Suppose f is some function and a is in the domain of f. If $f(a) > f(x)$ when x is near a, then a **relative maximum** occurs at $x = a$ and has **relative maximum value** $f(a)$. Very often, the function f *changes from increasing to decreasing* at a.

Suppose f is some function and b is in the domain of f. If $f(b) < f(x)$ when x is near b, then a **relative minimum** occurs at $x = b$ and has **relative minimum value** $f(b)$. Very often, the function f *changes from decreasing to increasing* at b.

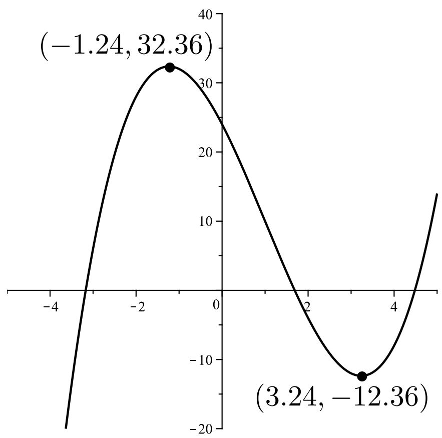
Example 2. In Example 1, a relative maximum occurs at $x = -3$ and has relative maximum value $f(-3) = 3$. In fact, f has *absolute maximum value* 3 at $x = -3$ because no point on the graph has a y-coordinate that is larger than 3.

A relative minimum occurs at $x = 1$ and has relative minimum value $f(1) = -2$, but this is not the *absolute minimum value* because there are points on the graph that have y-coordinates which are smaller than -2.

Example 3. For the function $f(x) = x^3 - 3x^2 - 12x + 24$, estimate the relative extrema, and use this information to determine the increasing, decreasing, or constant behavior of f.

Open the Graph program. Click on the [Insert function] button. Enter f as `x^3-3x^2-12x+24` and click [OK]. Click on the [Edit axes] button, or double-click "Axes", and change the values of x and y so that you can see the "humps" in the graph. Click on the [Evaluate] button and a menu will open. From the drop-down menu, beside where it says "Snap to:", select "Extremum".

Now, click on the graph somewhere near the relative maximum. You should see that the Graph program snaps to the relative maximum. The rel max occurs at $x \approx -1.24$ and has rel max value about 32.36. Again, click on the graph, but this time somewhere near the relative minimum. The rel min occurs at $x \approx 3.24$ and has rel min value about -12.36.



Since the function changes from increasing to decreasing at $x \approx -1.24$ and changes from decreasing to increasing at $x \approx 3.24$, the function increases on $(-\infty, -1.24) \cup (3.24, \infty)$ and decreases on $(-1.24, 3.24)$. The function exhibits no constant behavior.

Example 4. Try the following application problems from your book: pages 128–30, Exercises 23–33 odd. You will be expected to know basic plane geometry, such as how to find perimeter, area, and volume. Formulas for common figures are given in the back of your textbook.

Piecewise Functions

Piecewise functions use different output formulas over different parts of the domain. When graphed, piecewise functions often appear as several broken pieces.

Example 5. Graph

$$f(x) = \begin{cases} 4, & \text{for } x \leq -2, \\ x+1, & \text{for } -2 < x < 3, \\ -x, & \text{for } x \geq 3. \end{cases}$$

Find the domain and range of f and give the intervals of increasing, decreasing, or constant behavior.

The above set of symbols is the usual method for expressing a piecewise function. What it says is

- For values of x that are less than or equal to -2, the function f is defined as

$$f(x) = 4.$$

- For values of x between -2 and 3, the function f is defined as

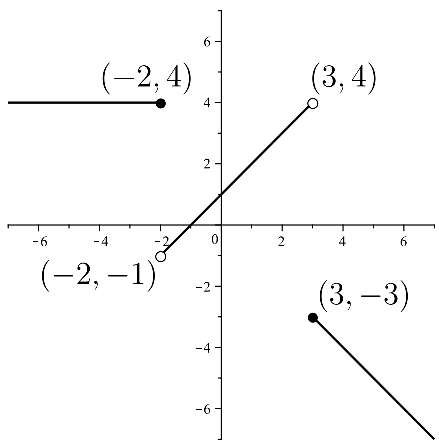
$$f(x) = x + 1.$$

- For values of x that are greater than or equal to 3, the function f is defined as

$$f(x) = -x.$$

As we can see, over different parts of the domain, the function f uses different output formulas.

The graph of f looks like several broken pieces:



domain: all reals $= (-\infty, \infty)$

range: $(-\infty, -3] \cup (-1, 4]$

increasing: $(-2, 3)$

decreasing: $(3, \infty)$

constant: $(-\infty, -2)$

Step functions are piecewise functions that have graphs which look like steps. One of the most common step functions is the **greatest integer function**, or *floor function*:

$$\begin{aligned} f(x) &= [\![x]\!] && \text{(one way to write)} \\ &= \lfloor x \rfloor && \text{(another way to write)} \\ &= \text{the greatest integer } \leq x && \text{(what it means)} \end{aligned}$$

Basically, if x is an integer, then $[\![x]\!] = x$. If x is not an integer, then $[\![x]\!]$ is the integer *to the left of* x on a number line.

Example 6. $[\![3.2]\!] = 3$, $[\![14.9]\!] = 14$, $[\![\frac{\sqrt{7}}{2}]\!] = 1$, $[\![22]\!] = 22$, and $[\![-\pi]\!] = -4$.