Section 1.4: Equations of Lines and Modeling

Key points:

- Use the slope-intercept form of a line.
- Use the point-slope form of a line.
- Determine whether two lines are are parallel or whether they are perpendicular.

Slope-Intercept Form of a Line

The **slope-intercept form** of a line is given by

$$f(x) = mx + b \quad \text{or} \quad y = mx + b,$$

where m is the slope of the line and the point $(0, b)$ is the y-intercept.

Example 1. Find the slope and y-intercept of $y = 2x - 4$.

Since this line is already written in slope-intercept form, the answer is easy. The slope of the line is $m = 2$ and the y-intercept is the point $(0, -4)$.

Example 2. Find the slope and y-intercept of $3x + 5y - 7 = 0$.

In order to solve this problem, we need to write the given equation in slope-intercept form, which really means we *just need to solve for* y:

$$\begin{aligned} 3x + 5y - 7 &= 0 \\ 5y &= -3x + 7 \\ y &= -\frac{3}{5}x + \frac{7}{5} && \text{(slope-int form)} \end{aligned}$$

The slope is $m = -\frac{3}{5}$ and the y-intercept is the point $(0, \frac{7}{5})$.

Point-Slope Form of a Line

The **point-slope form** of a line is given by

$$y - y_1 = m(x - x_1),$$

where m is the slope of the line and (x_1, y_1) is any point on the line.

Example 3. Find the equation of the line through $(-1, 5)$ and $(3, -4)$ in slope-intercept form.

Since we are given two points on the line, we can find the slope:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{3 - (-1)} = \frac{-9}{4}.$$

Now that we have found the slope, we can use the point-slope form of a line to write an equation for the line:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= -\frac{9}{4}(x - (-1)) && \text{(substitution)} \\ y - 5 &= -\frac{9}{4}(x + 1). && \text{(pt-slope form)} \end{aligned}$$

Since we want the slope-intercept form of this line, we just need to solve this last equation for y:

$$\begin{aligned} y - 5 &= -\frac{9}{4}x - \frac{9}{4} && \text{(after distributing)} \\ y &= -\frac{9}{4}x - \frac{9}{4} + 5 \\ y &= -\frac{9}{4}x + \frac{11}{4}. && \text{(slope-int form)} \end{aligned}$$

Parallel and Perpendicular Lines

Two lines are **parallel** when they have the *same slope* and *different intercepts*. If two lines have the same slope and intercepts the the lines are the same line, and we do not consider them to be different.

Note 1: The vertical lines $x = 3$ and $x = -2$ are parallel. They both have undefined slope and different x-intercepts.

Note 2: The horizontal lines $y = 1$ and $y = 4$ are parallel. They both have slope $m = 0$ and different y-intercepts.

Two lines are **perpendicular** when their *slopes are negative reciprocals*, that is, when the product of their slopes is -1, and the lines meet at a right angle. When we say "negative reciprocal," we mean "flip and change the sign."

Note 3: The horizontal line $y = b$ is perpendicular to the vertical line $x = a$. Every vertical line is perpendicular to every horizontal line.

Example 4. Find the equations of the lines in slope-intercept form that are (a) parallel to and (b) perpendicular to the line $-2x + 4y = 6$ through the point $(3, 1)$. (c) Graph all 3 lines on the same set of axes.

First, we need to find the slope of the given line by solving for y:

$$\begin{aligned} -2x + 4y &= 6 \\ 4y &= 2x + 6 \\ y &= \frac{1}{2}x + \frac{3}{2}, && \text{(slope-int form)} \end{aligned}$$

so the slope of the given line is $m = \frac{1}{2}$.

(a) The equation of the line we want has the same slope as the given line but goes through $(3, 1)$, so using the point-slope form of a line, we find:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= \frac{1}{2}(x - 3), && \text{(pt-slope form)} \end{aligned}$$

which we can convert to slope-intercept form by solving for y:

$$\begin{aligned} y - 1 &= \frac{1}{2}x - \frac{3}{2} && \text{(after distributing)} \\ y &= \frac{1}{2}x - \frac{3}{2} + 1 \\ y &= \frac{1}{2}x - \frac{1}{2} && \text{(slope-int form)} \end{aligned}$$

(b) The equation of the line we want has slope which is the negative reciprocal of the given line, which would be -2, but goes through $(3, 1)$. Again, using the point-slope form, we find

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -2(x - 3), && \text{(pt-slope form)} \end{aligned}$$

which we can convert to slope-intercept form by solving for y:

$$y - 1 = -2x + 6 \qquad \text{(after distributing)}$$

$$y = -2x + 7 \qquad \text{(slope-int form)}$$

(c) The graphs of the 3 lines are given below: