Section 1.3: Linear Functions, Slope, and Applications

Key points:

- Given a linear function, find the slope, the x-intercept, and the y-intercept of the graph, if possible, and *vice versa*.
- Find the coordinates of points on a line.
- Know the difference between a constant function (horizontal line), the identity function, and a vertical line.
- Understand how the slope of a line affects the behavior of its graph.
- Find the slope of a line given two points on the line.
- Interpret and use grade.
- Find, interpret, and use the average rate of change of a function.
- Solve applied problems involving slope, grade, and average rate of change.

Linear Functions

Linear Function: A polynomial function of degree at most 1; a linear function "looks like"

$$f(x) = mx + b \qquad \text{(linear function)},$$

where m is the **slope** of the line and the point $(0, b)$ is the y-intercept of the graph. The numbers m and b are both real numbers.

Example 1. Suppose $g(x) = 5x + 2$. Then g is a linear function with slope $m = 5$ and y-intercept $(0, 2)$. We can find several points on g by plugging in various values of x:

x	$g(x)$
-2	-8
-1	-3
1	7
2	12

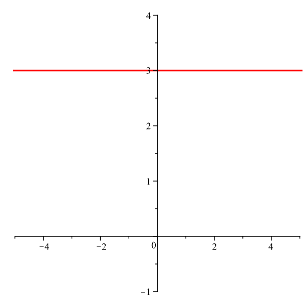
Exercise 1. Find the linear function h with slope $m = -\dfrac{1}{2}$ and y-intercept $(0, 3)$. Find the coordinates of the x-intercept.

Exercise 2. Find the linear function f with slope $m = 3$ that goes through the point $(-2, 0)$. Find the coordinates of the y-intercept.

Constant Function. A linear function with slope $m = 0$ and y-intercept $(0, b)$. The equation "looks like"

$$f(x) = b \qquad \text{(constant function).}$$

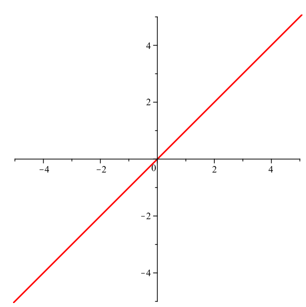
A constant function is a **horizontal line**. No matter what value you put in for x, you always get the same value of y out.



Identity Function. A linear function with slope $m = 1$ and y-intercept $(0, 0)$. The identity function is

$$f(x) = x \qquad \text{(identity function).}$$

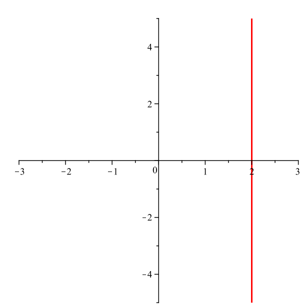
Basically, whatever you put in is what you get out. The origin is also the x-intercept.



Vertical Line. The *only* type of line that is **not a function**. The slope is **undefined** and the equation "looks like"

$$x = a \qquad \text{(vertical line),}$$

where a is a real number. The x-intercept is $(a, 0)$.



slope		"looks like"	behavior	function?
positive	$m > 0$	$\nearrow$	increasing	yes
negative	$m < 0$	$\searrow$	decreasing	yes
zero	$m = 0$	$\longleftrightarrow$	constant (horiz.)	yes
undefined	$m = \text{und}$	$\updownarrow$	n/a (vert.)	no

Slope. The slope of a linear function f can be expressed as

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{\Delta f}{\Delta x} = \frac{\text{change in } f}{\text{change in } x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}, \end{aligned}$$

where Δ is read as "delta" and usually means "change in" in mathematics.

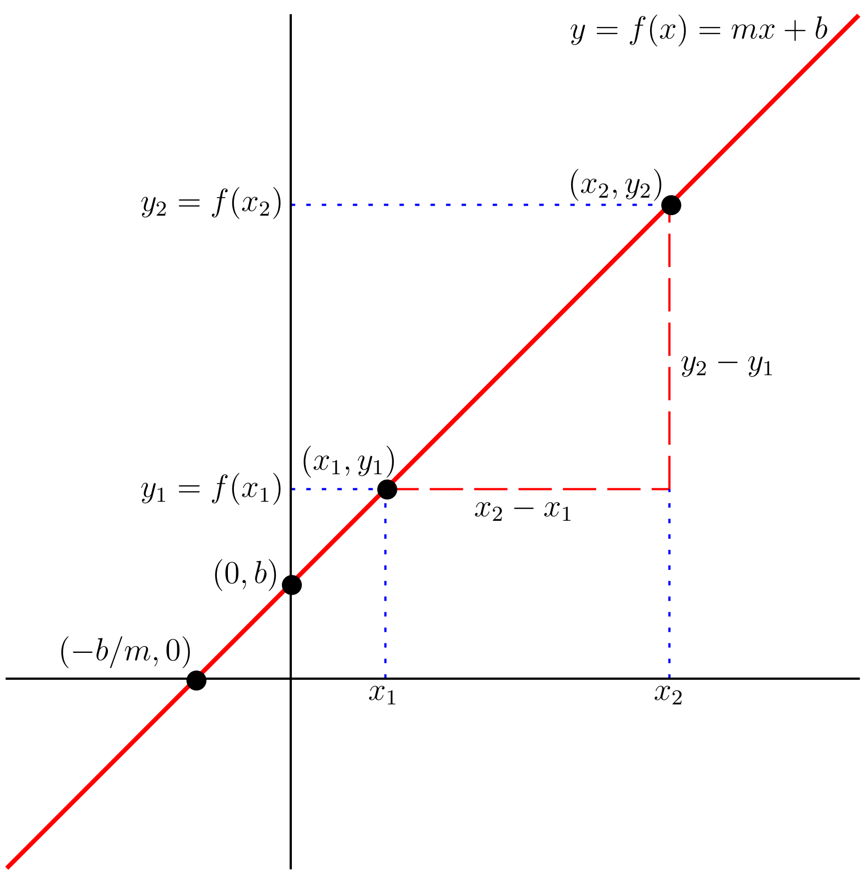


Figure 1: Note that this particular line has non-zero slope; in fact, it has positive slope.

Exercise 3. Find the slope, x-intercept, and y-intercept, if possible. If the graph is the graph of a function, describe the graph as either increasing, decreasing, or constant. If the graph is not the graph of a function, then state so.

(a) $f(x) = -3x + 5$

(b) $g(x) = 2.3\pi$

(c) $h(x) = \frac{1}{4}x + 2$

(d) $x = -1$

Exercise 4. Find the equation of the linear function that goes through the points $(0, 3)$ and $(2, -1)$. Find the x-intercept and describe the function as either increasing, decreasing, or constant.

Exercise 5. Find the equation of the linear function that goes through the points $(-4, -2)$ and $(-1, 3)$. Find the x-intercept, the y-intercept, and describe the function as either increasing, decreasing, or constant.

Grade. Grade is a type of slope–it is the ratio of the change in height (rise) over some horizontal distance (run). It is a measure of the *steepness* of an incline or decline.

Example 2. A 4% grade means that the height increases 4 units for every 100 units increase in horizontal distance; for example,

$$4\% \text{ grade} = \frac{4 \text{ ft}}{100 \text{ ft}} = \frac{4 \text{ in}}{100 \text{ in}} = \frac{4 \text{ mi}}{100 \text{ mi}} = \frac{1}{25}$$

Average Rate of Change of a Function

The **average rate of change** of a function f between two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the *secant line* through those points; that is,

$$\text{average rate of change of } f = \frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

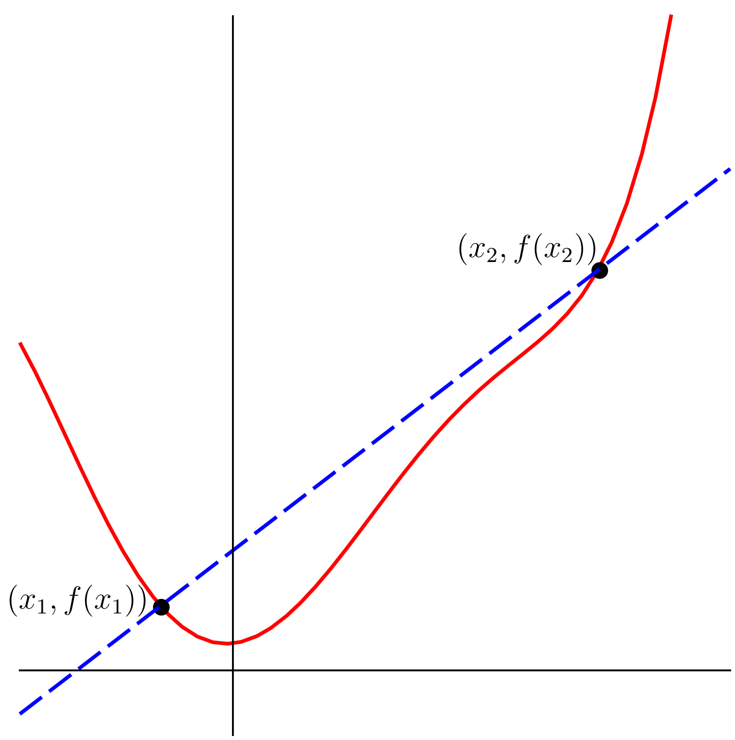


Figure 2: This figure shows the graph of some function $y = f(x)$ (solid) and the secant line (dashed) through two points on the graph of f.

Exercise 6. Solve: Problem #44, p. 100.

Exercise 7. Solve: Problem #48, p. 101.

Exercise 8. Solve: Problem #53, p. 101-2.

Exercise 9. Solve: Problem #56, p. 102.