

ODD-GRACEFUL TREES

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A graph G of size n is *odd-graceful* if there is an injection f from $V(G)$ to $\{0, 1, 2, \dots, 2n - 1\}$ such that, when each edge xy is assigned the label or *weight* $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, 5, \dots, 2n - 1\}$. This definition was introduced by Gnanajothi [4] in 1991.

In this note we survey the results related with trees.

In [4], Gnanajothi proved that every graph with an α -labeling has an odd graceful labeling. For a detailed account of α -labelings the reader is referred to [3]. Among the several families of odd-graceful trees found by Gnanajothi, two are interesting because they are not in the class of α -graphs:

- rooted trees of height 2
- the trees obtained from a star by adjoining to each end vertex the path P_3 or by adjoining to each end vertex the path P_4 .

She also proved that all trees with order up to 10 are odd-graceful; she conjectures that all trees are odd graceful.

Eldergill [2] proved that the trees obtained joining one end vertex from each of any number of paths of equal length is odd-graceful.

Sekar [5] proved that lobsters, banana trees, and regular bamboo trees are odd-graceful.

Yao, Cheng, Zhongfu, and Yao [6] have shown the following: if a tree T has an edge u_1u_2 such that the two components T_1 and T_2 of $T - u_1u_2$ have the properties that $d_{T_1}(u_1) \geq |T_1|/2$ and T_2 is a caterpillar, then T is odd graceful; if a tree T has a vertex of degree at least $|T|/2$, then T is odd graceful.

In [1] we have proved that all trees of diameter 5 are odd graceful. Note that trees of diameter at most 3 are caterpillars, so they have an α -labeling which can be used to find a suitable odd-graceful labeling. Trees of diameter 4 we studied in the case of rooted trees of height 2. We also proved that any forest whose components are caterpillars is odd-graceful; moreover, if we replace the condition of being a caterpillar by the condition of being an α -tree, the result also holds.

Here we show odd-graceful labelings for all trees of diameter greater than 5 that are not caterpillars.

In addition we generalize one of Gnanajothi's results; we show that the tree obtained by attaching to each vertex of the path P_n the path P_k is odd-graceful. This tree is an α -tree, so the result is obvious however we have decided to show the construction used (see Figure 1).

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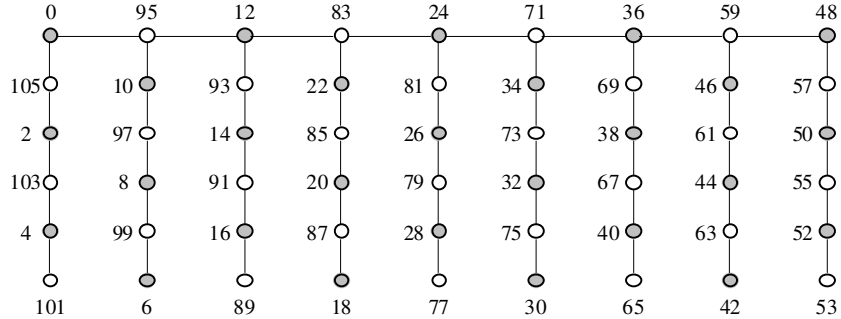


Fig. 1. Odd-graceful labeling of a comb

If any number of horizontal edges is moved down, the resulting tree is also odd-graceful. We show an example in Figure 2.

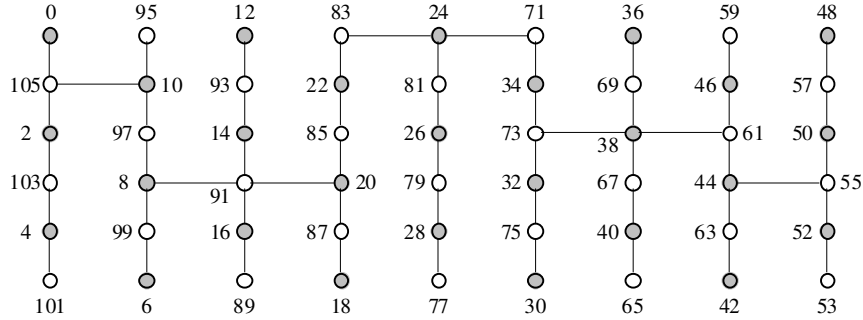


Fig. 2. Odd-graceful tree

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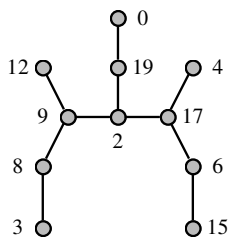
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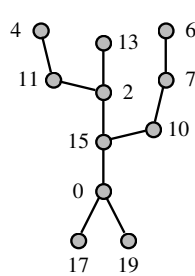
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Odd-graceful graphs of order 11

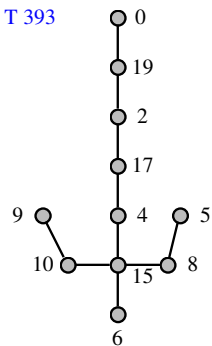
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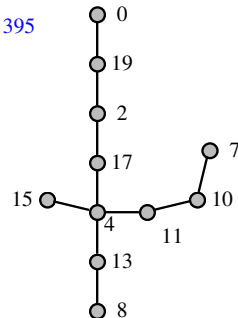
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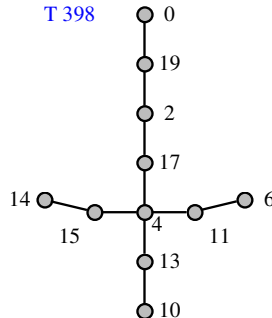
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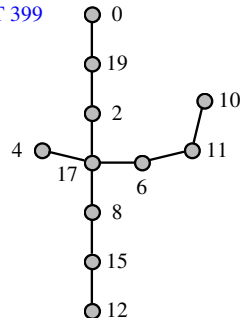
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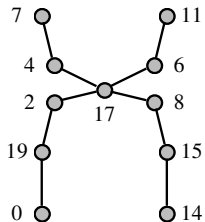
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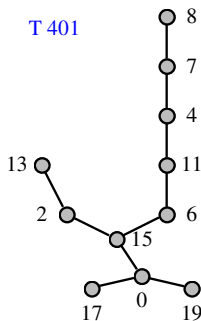
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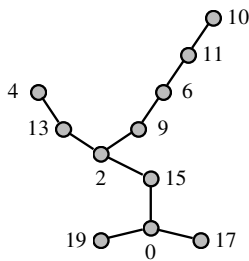
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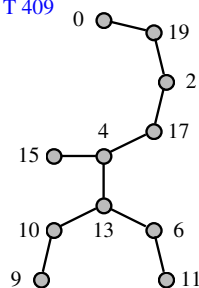
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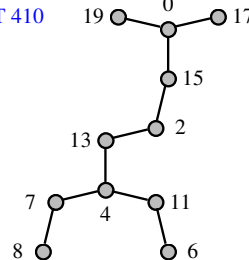
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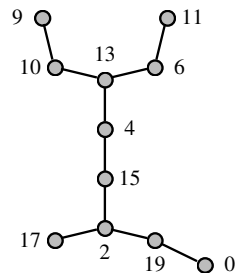
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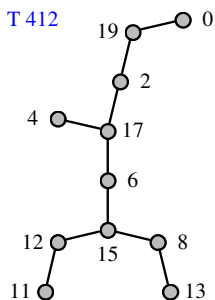
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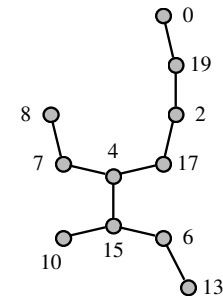
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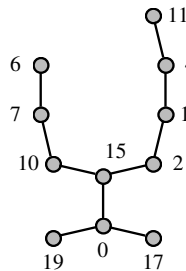
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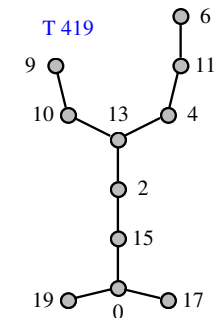
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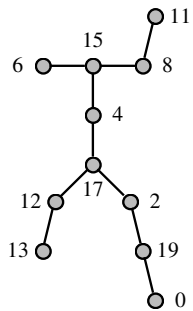
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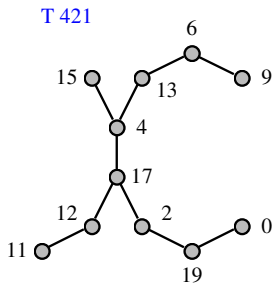
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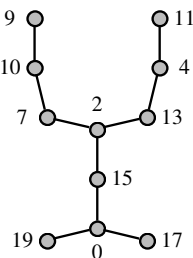
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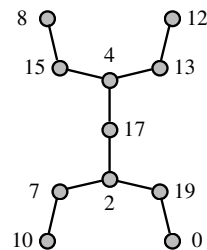
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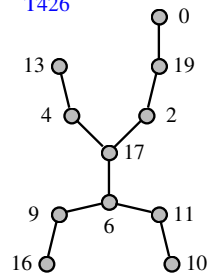
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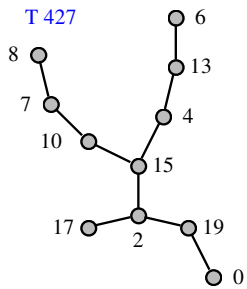
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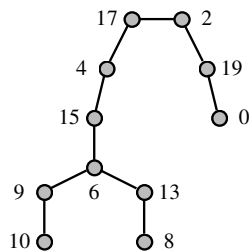
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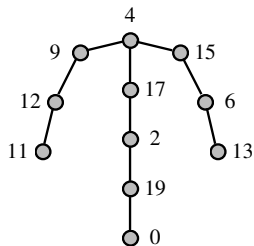
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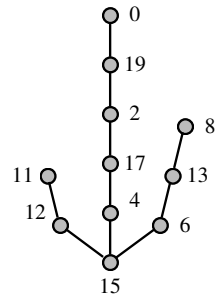
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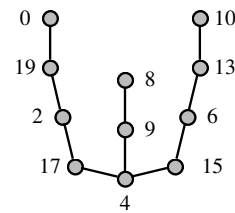
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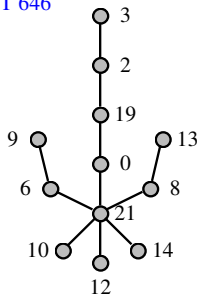


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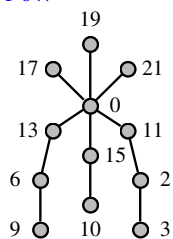


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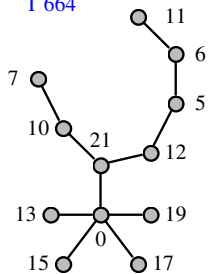
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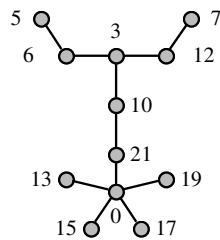
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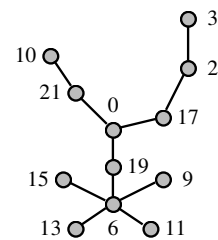
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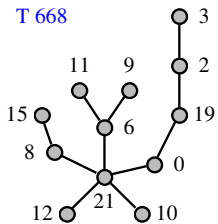
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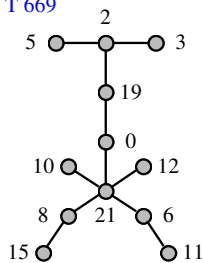
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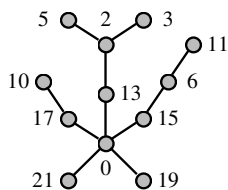
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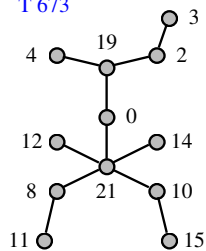
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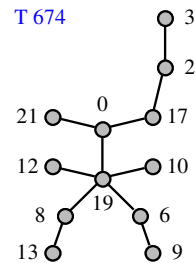
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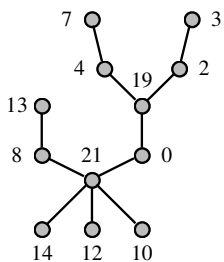
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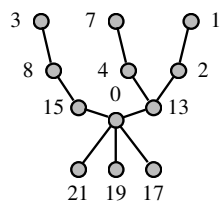
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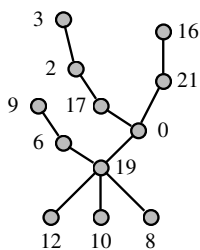
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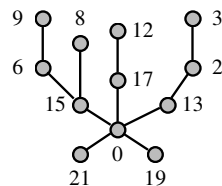
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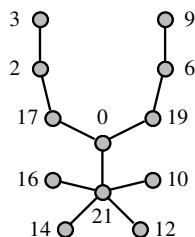
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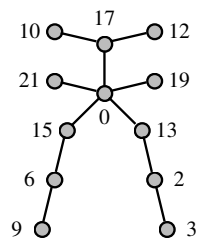
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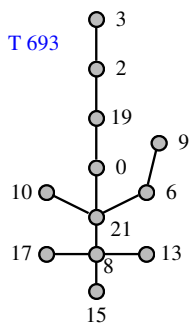
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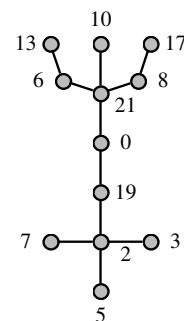
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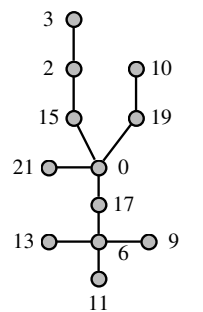
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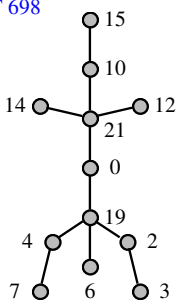
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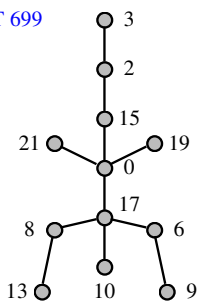
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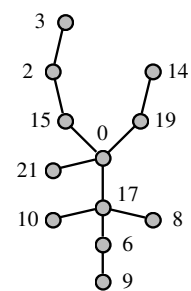
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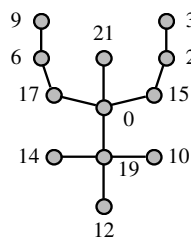
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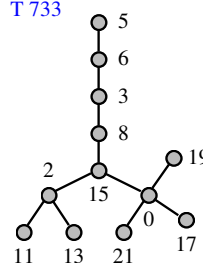
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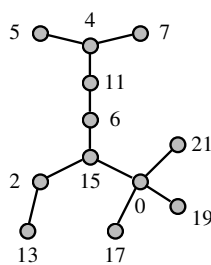
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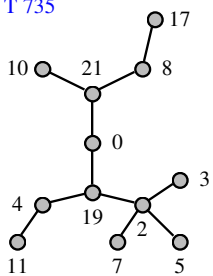


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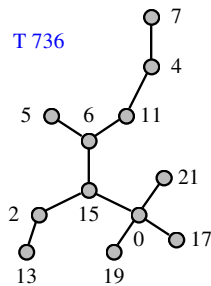


Odd-graceful graphs of order 12

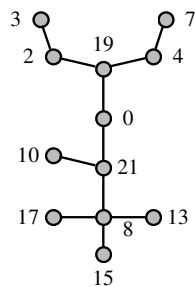
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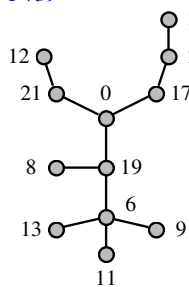
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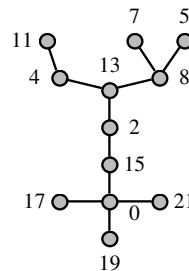
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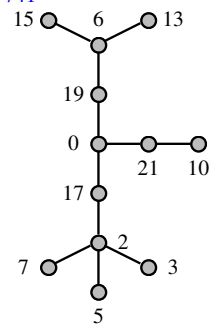
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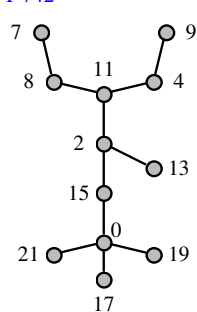
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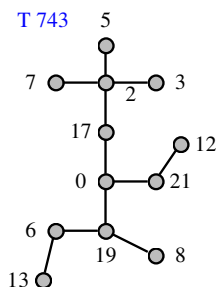
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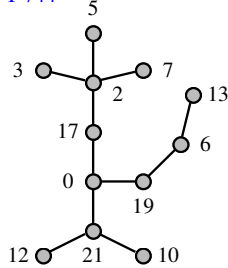
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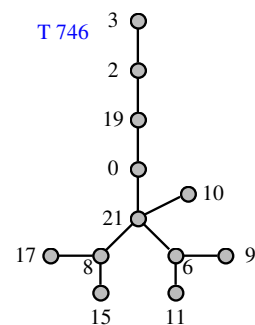
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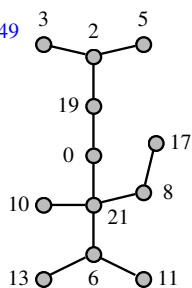
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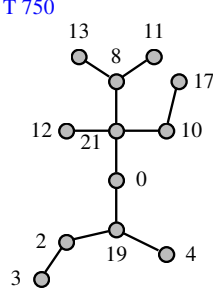
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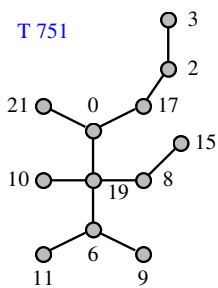
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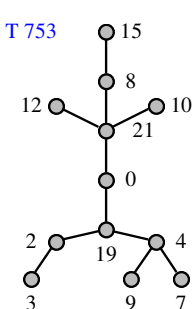
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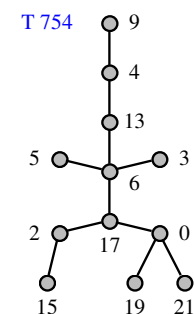
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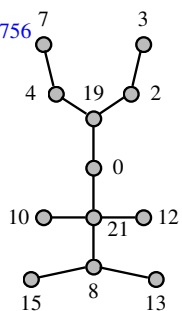
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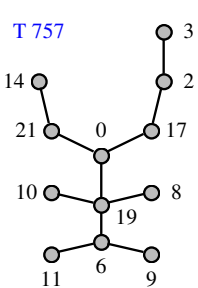
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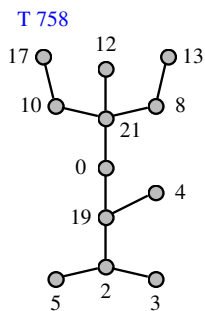
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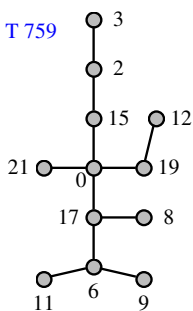
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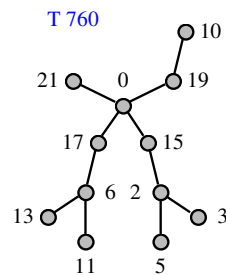
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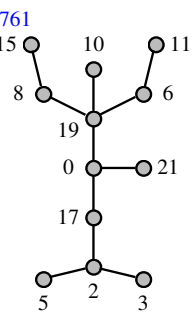
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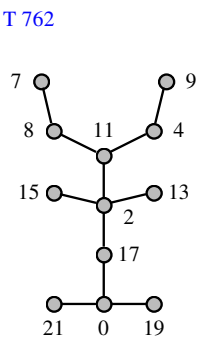
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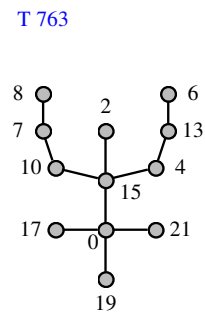
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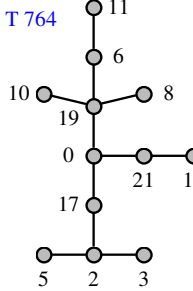
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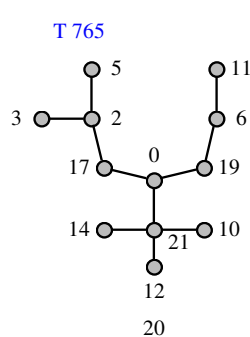
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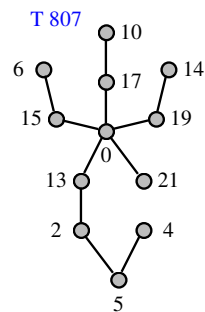
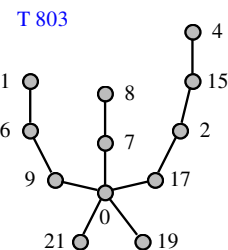
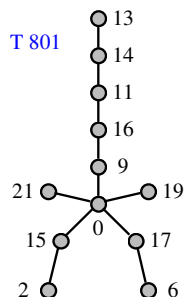
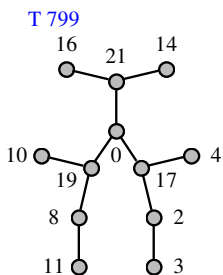
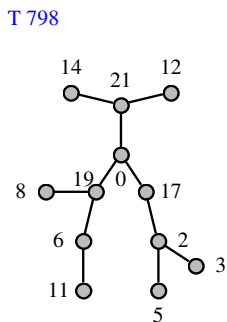
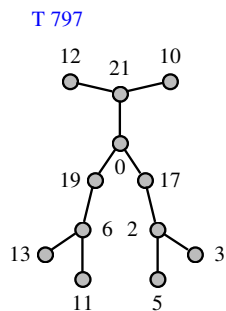
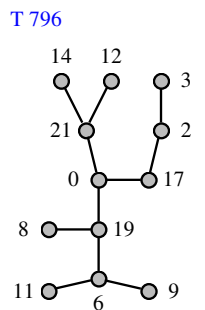
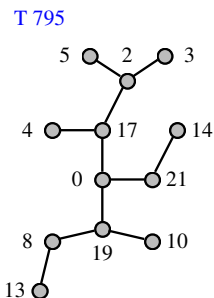
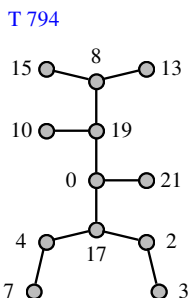
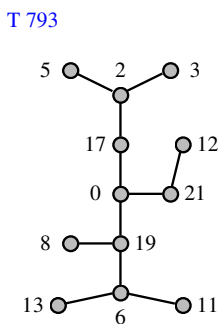
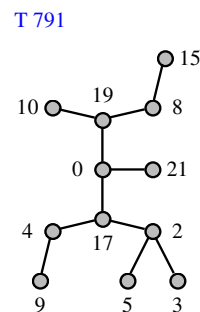
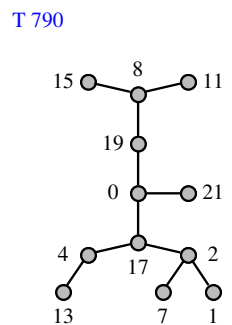
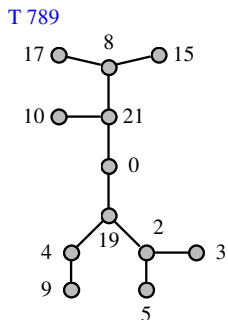
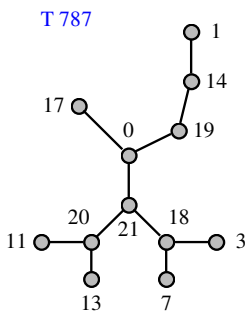
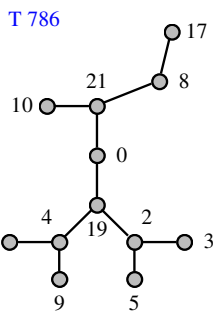
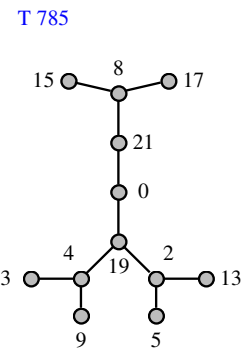
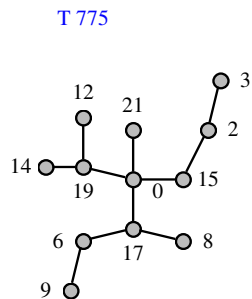
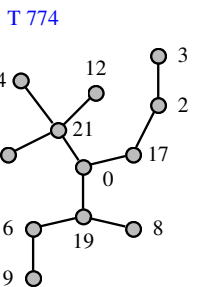
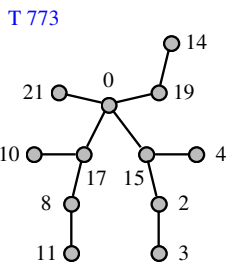
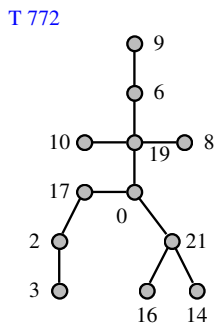
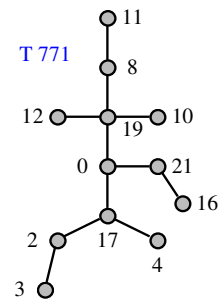
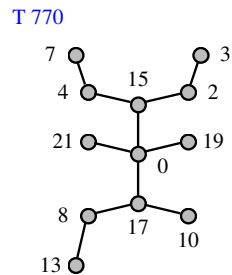
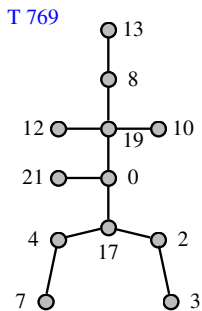
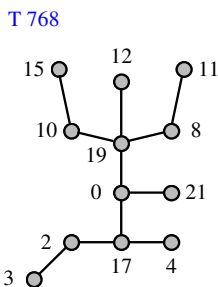
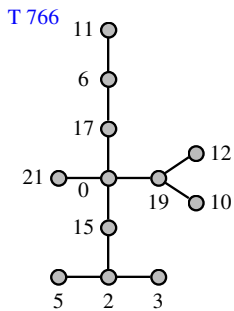
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T 765

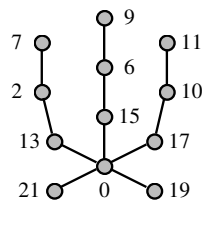


Odd-graceful graphs of order 12

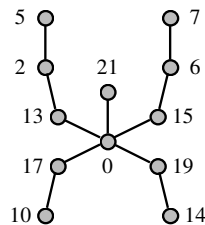


Odd-graceful graphs of order 12

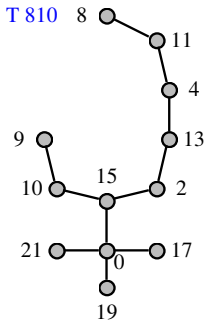
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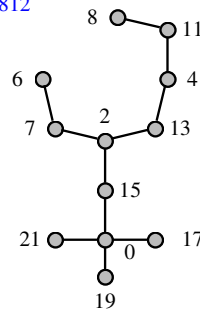
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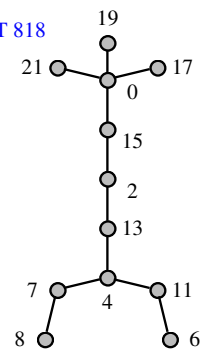
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T 812

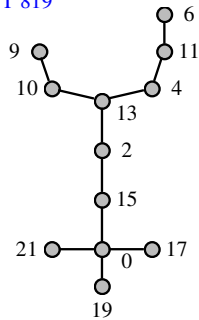


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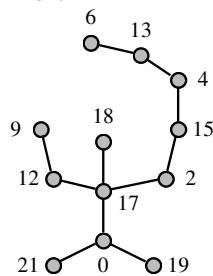


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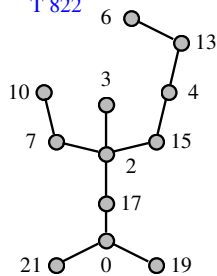
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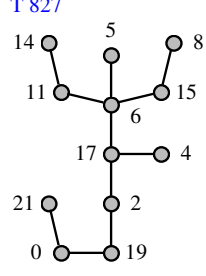
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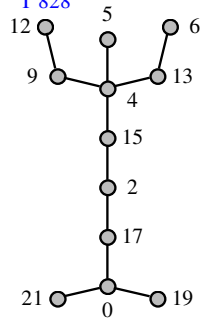
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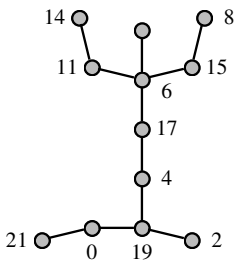
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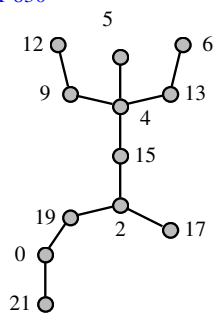
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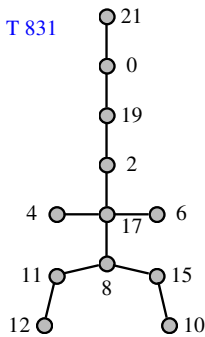
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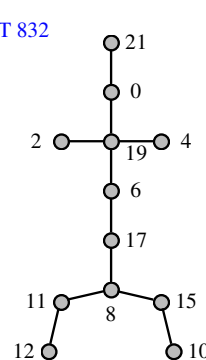
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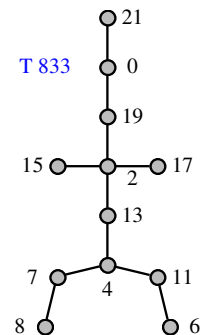
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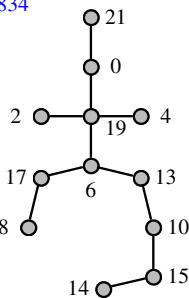
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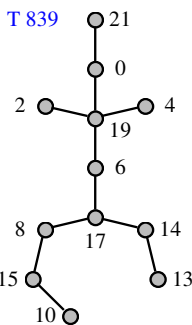
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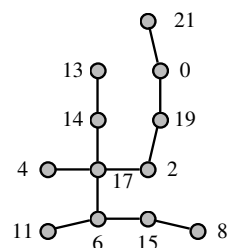
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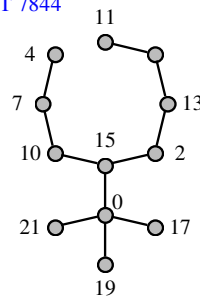
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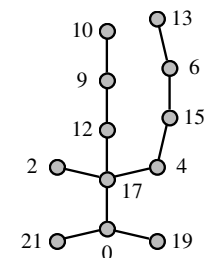
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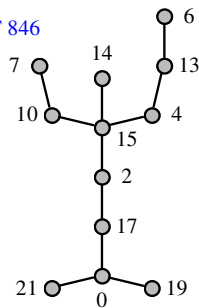
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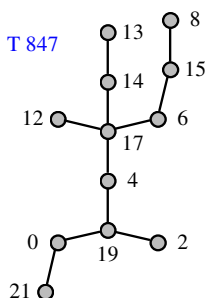
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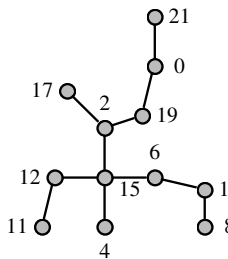
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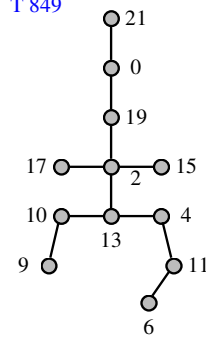
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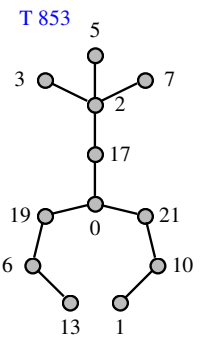
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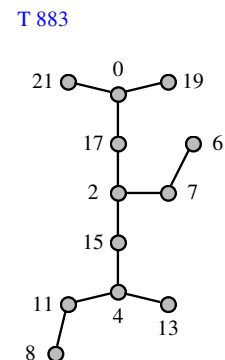
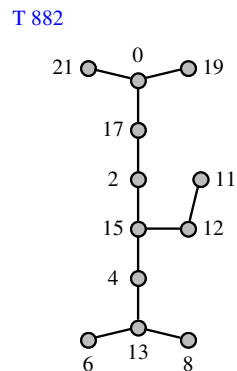
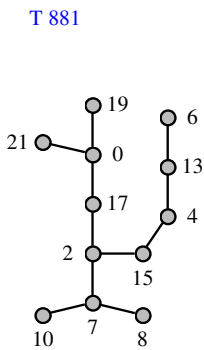
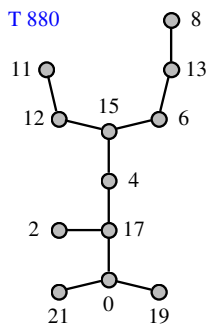
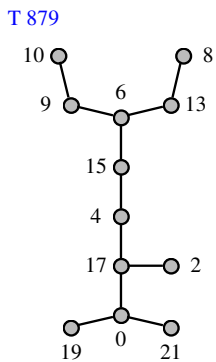
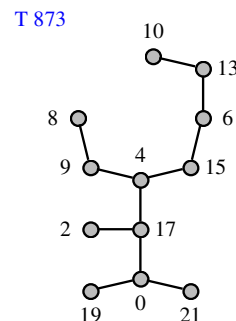
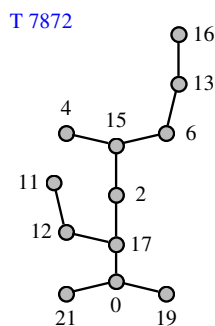
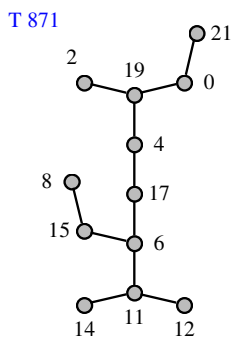
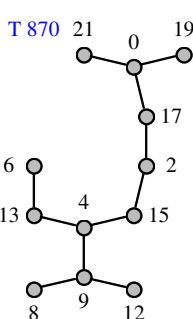
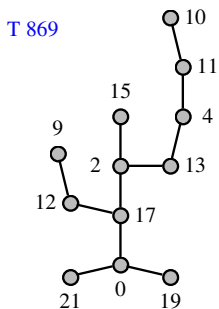
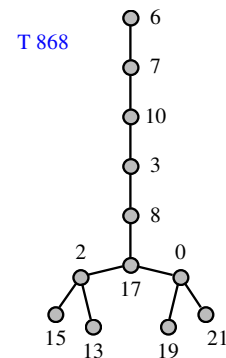
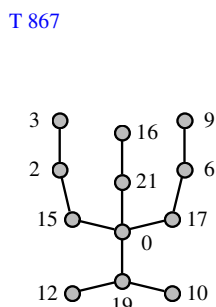
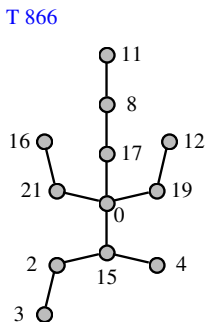
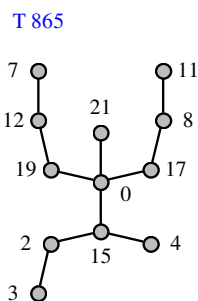
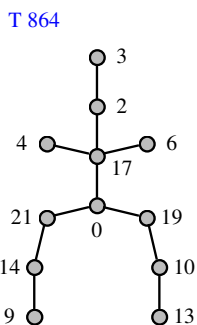
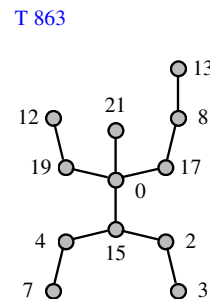
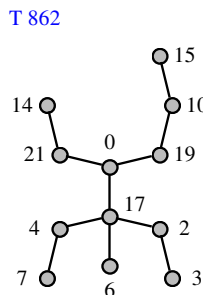
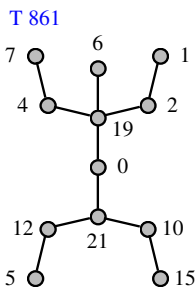
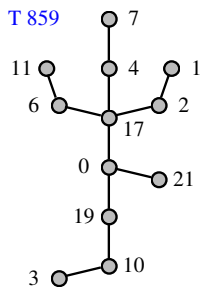
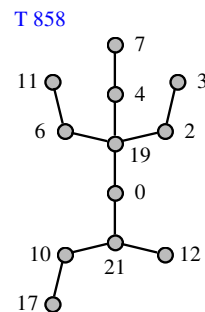
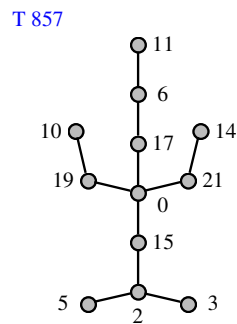
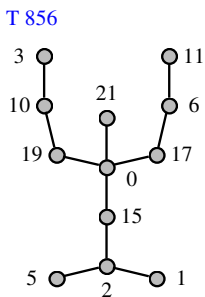
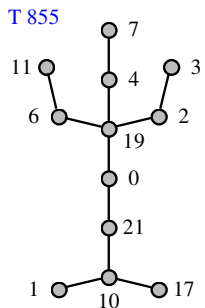
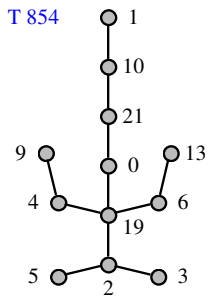
T 849



T 853

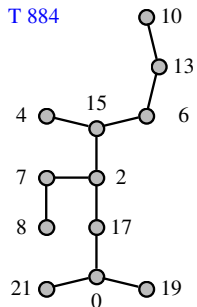


Odd-graceful graphs of order 12

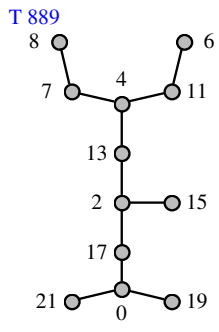


Odd-graceful graphs of order 12

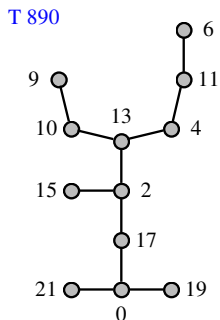
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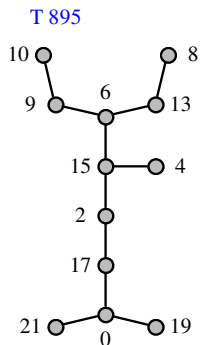
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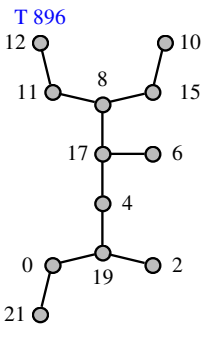
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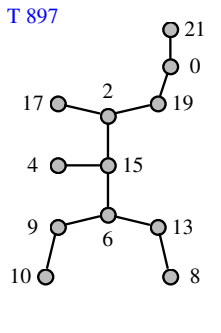
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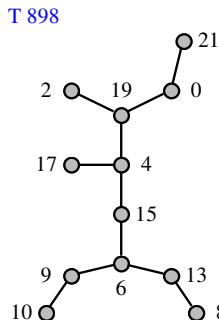
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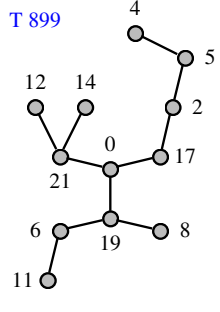
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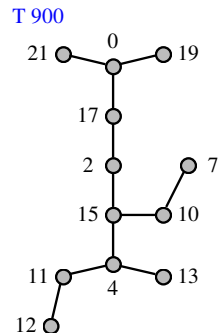
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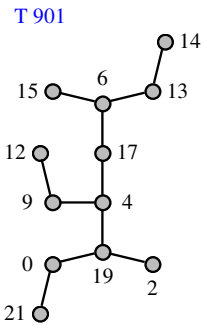
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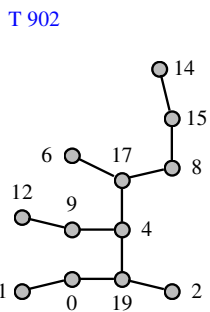
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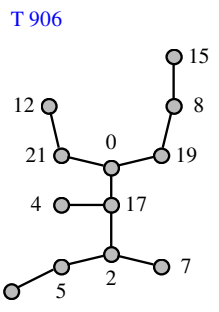
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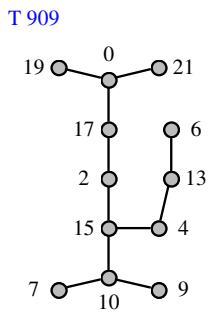
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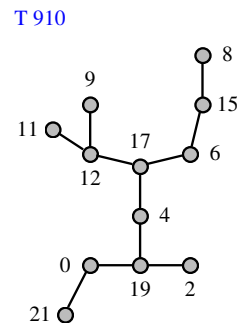
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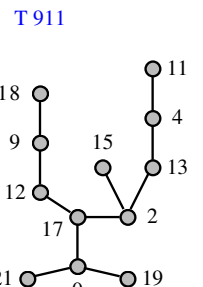
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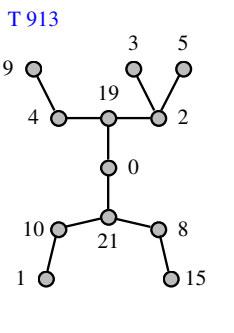
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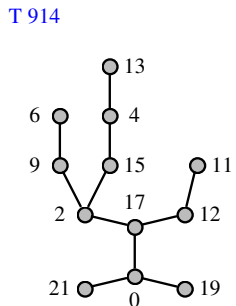
T 911



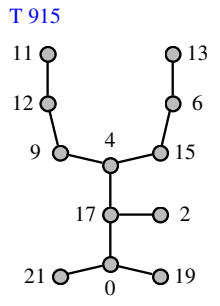
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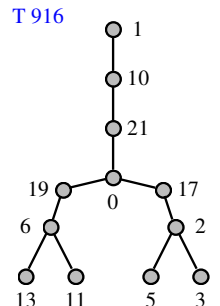
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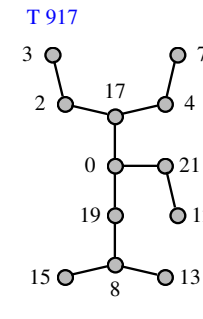
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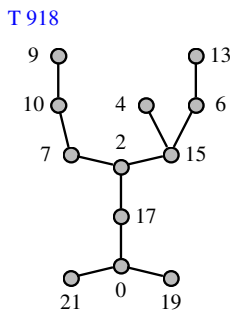
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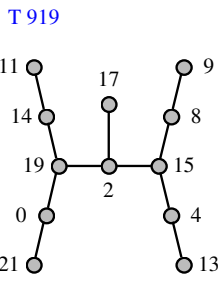
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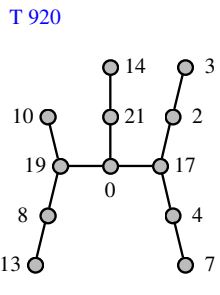
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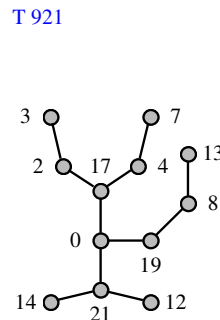
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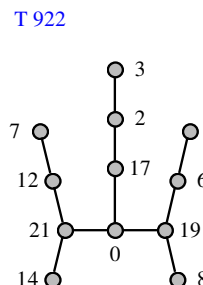
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T 921

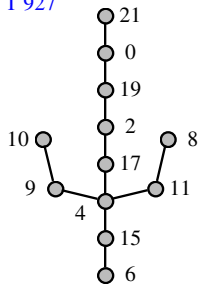


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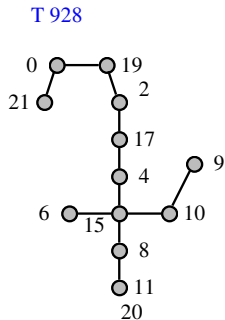


Odd-graceful graphs of order 12

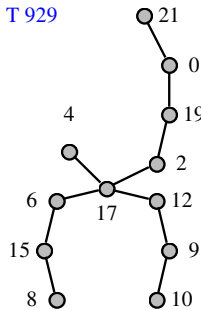
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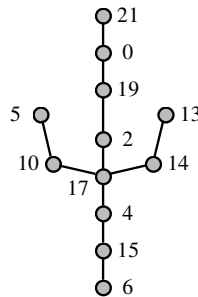
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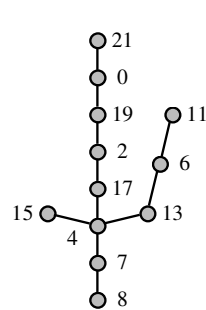
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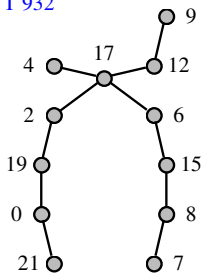
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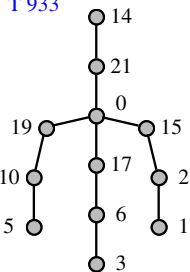
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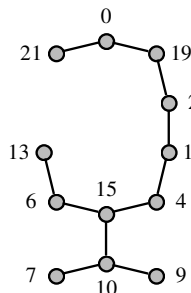
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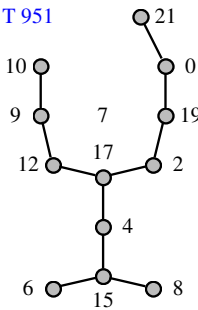
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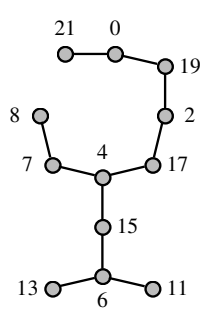
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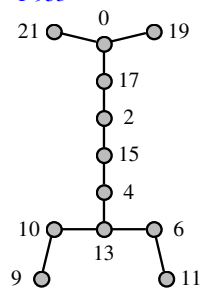
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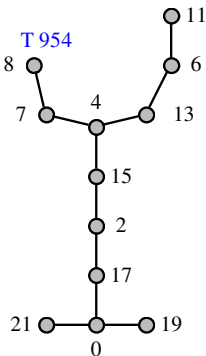
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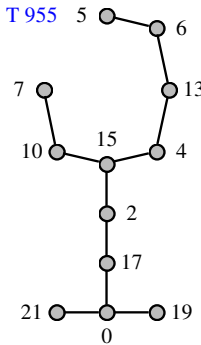
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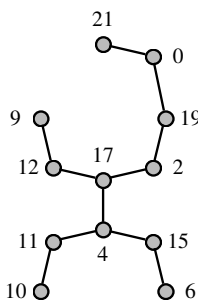
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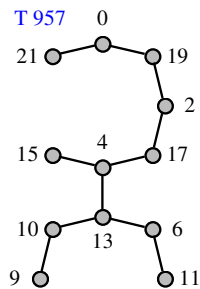
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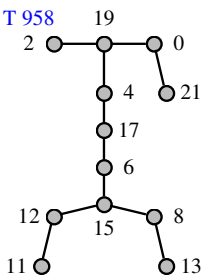
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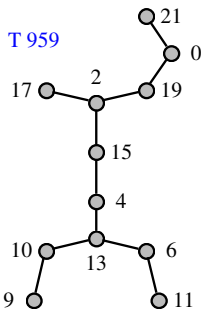
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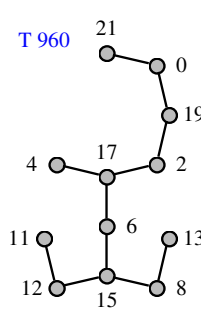
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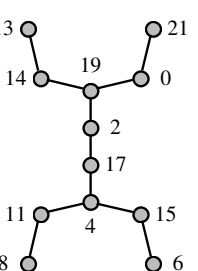
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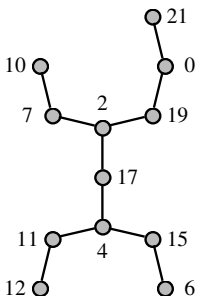
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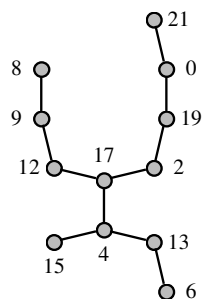
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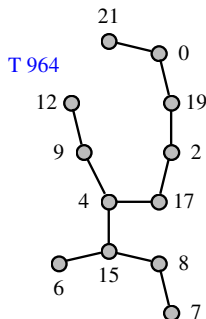
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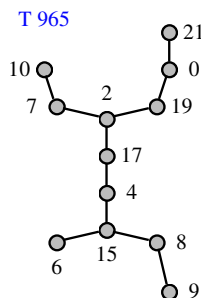
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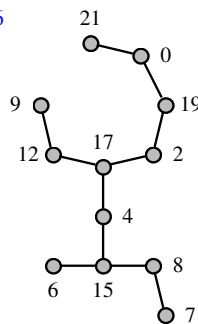
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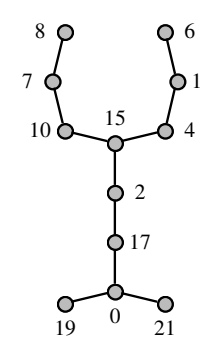
T 965



T 966



T 967



Odd-graceful graphs of order 12

